Note Title 9/29/2019

ENEE 610
Tu 10/01/19

Lossless 1-port synthesis: via $z(s)$ &or $y(s)$ ↳ PR

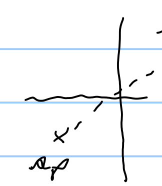
1st Foster, $z(s)$ } partial
2nd Foster, $y(s)$ } fractions

1st Cauer, @ $s=\infty$ } continued
2nd Cauer; @ $s=0$ } fractions

$y(s)$ is odd $\Rightarrow$ $\frac{y(s)+y(-s)}{2} = \mathrm{Ev}\, y(s) = 0$, $y(s) = -y(-s)$ lossless

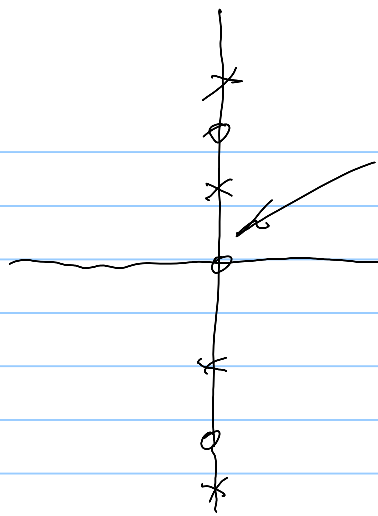
PR $\Rightarrow$ real coefficients, no poles in $\sigma > 0$

$\Rightarrow$ all poles on $j\omega$ axis



$-s_p$ = pole of $y(-s) = y(s)$
$\Rightarrow$ a pole of $y(s)$
but can't be

PR then means poles and zeros alternate on $j\omega$ axis, are simple,
& residues are real and >0



@ $s=0$ a pole or a zero

Ex: $y(s) = \dfrac{s(s^2+5)}{(s^2+3)}$ $\qquad \delta = \text{degree}[y(s)] = 3$

$j = \sqrt{-1}$

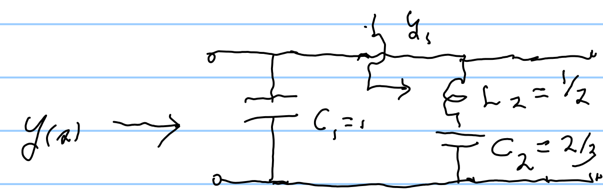
$y(s) = k_\infty \cdot s + \dfrac{k_{\sqrt{3}}}{s+j\sqrt{3}} + \dfrac{k_{-\sqrt{3}}}{s-j\sqrt{3}}$ $\qquad k_\infty \sim \dfrac{s^3}{s^2} \times k_\infty \Rightarrow k_\infty = 1$

for $k_{\sqrt{3}}$ $\times (s+j\sqrt{3}) \Rightarrow \dfrac{s(s^2+5)}{(s+j\sqrt{3})(s-j\sqrt{3})} \times (s+j\sqrt{3}) = \dfrac{s(s^2+5)}{s-j\sqrt{3}} = k_\infty s(s+j\sqrt{3}) + k_{\sqrt{3}} + \dfrac{k_{-\sqrt{3}}(s+j\sqrt{3})}{s-j\sqrt{3}}$

set $s = -j\sqrt{3} \Rightarrow \dfrac{-j\sqrt{3}(-3+5)}{-j2\sqrt{3}} = \dfrac{2}{2} = 1 = 0 + k_{\sqrt{3}} + 0 \Rightarrow k_{\sqrt{3}} = 1$

$y(s) = 1 \cdot s + \dfrac{1}{s+j\sqrt{3}} + \dfrac{k_{-\sqrt{3}}}{s-j\sqrt{3}}$ $\qquad k_{-\sqrt{3}} = k_{\sqrt{3}}^* = 1$

$$y(s) = s + \frac{(s-j\sqrt{3}) + (s+j\sqrt{3})}{s^2+3} = s + \frac{2s}{s^2+3} \Rightarrow \text{2nd Foster form}$$



$$y_1 = \frac{2s}{s^2+3} = \frac{1}{\frac{s}{2} + \frac{3}{2s}}$$

1st Foster: partial fraction expansion of $Z(s) = 1/y(s)$

$$Z(s) = \frac{s^2+3}{s(s^2+5)} = \frac{k_0}{s} + \frac{2k_{\sqrt{5}}\, s}{s^2+5}$$

for k_0 $\times s$: $\left.\frac{s^2+3}{s^2+5}\right|_{s=0} = \frac{3}{5} = k_0$

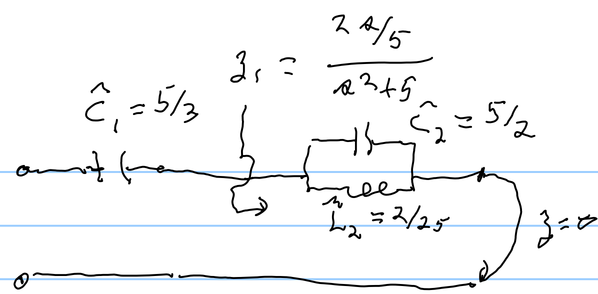
for $k_{\sqrt{5}} \Rightarrow \times\, s^2+5$: $\frac{s^2+3}{s} = \frac{k_0(s^2+5)}{s} + \frac{2k_{\sqrt{5}}\, s\,(s^2+5)}{(s^2+5)}$

$\div s \Rightarrow \left.\frac{s^2+3}{s^2}\right|_{s^2=-5} = 2k_{\sqrt{5}} = \frac{-5+3}{-5} = 2/5$

$$Z(s) = \frac{3}{5s} + \frac{2/5\, s}{s^2+5} + 0$$

$y(s) \rightarrow$

$= 1/z(s)$



1st Foster

$z_1 = \frac{2s/5}{s^2+5} = \frac{1}{\frac{5s}{2} + \frac{25}{2s}} \Big\} y.$

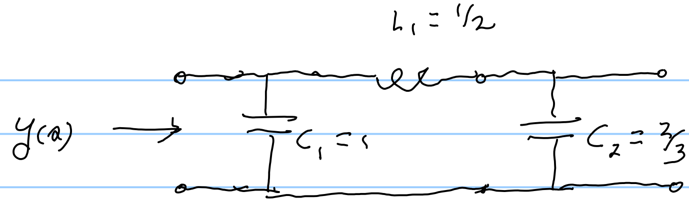
Cauer: 1st remove poles @ ∞

$y(s) = \frac{s(s^2+5)}{s^2+3}$ has $s = \infty$ as a poles

$= k_\infty s + \dots$

$y - k_\infty s$ is still LPR but no pole @ ∞

$$
\begin{array}{l}
s^2+3 \,\big)\, s^3 + 5s \;\; (s \quad \leftarrow s\,y \\
\qquad\quad\;\; s^3 + 3s \\
\qquad\quad\;\; \overline{\quad 2s \;\big)\, s^2 + 3 \;\; (\tfrac{1}{2}s \quad \leftarrow s\,z} \\
\qquad\qquad\qquad\quad s^2 \\
\qquad\qquad\qquad\quad \overline{\;\; 3 \;\big)\, 2s \;\; (2s/3 \quad \leftarrow s\,y} \\
\qquad\qquad\qquad\qquad\quad\;\; 2s \\
\qquad\qquad\qquad\qquad\quad\;\; \overline{\;\; 0} \quad \leftarrow s\,y
\end{array}
$$



1st Cauer

2nd Cauer remove poles @ 0

$y(s) = \dfrac{s(s^2+5)}{s^2+3}$ no pole @ 0 but one in z

$z(s) = \dfrac{s^2+3}{s^3+5s}$

(lowest powers of s)

$z \longrightarrow \frac{3}{5s}$

$5s + s^3 \,\overline{)\, 3 + s^2}$

$3 + \frac{3}{5}s^2$

$y \longrightarrow \frac{25}{2s}$

$\frac{2}{5}s^2 \,\overline{)\, 5s + s^3}$

$5s$

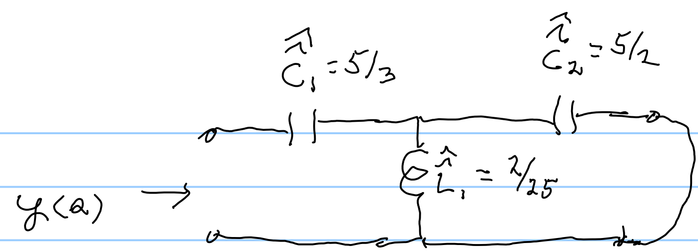
$z \longrightarrow \frac{2}{5s}$

$s^3 \,\overline{)\, \frac{2}{5}s^2}$

$\frac{2}{5}s^2$

$z \longrightarrow$

0



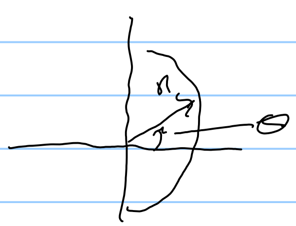
2nd Cauer.

are used $\delta[y(s)]$ reactive elements, minimal # of these.

these are the classical lossless 1-port designs of an LPR y or z

Reason for simple poles in $y(s)$, PR, on $j\omega$ axis

$$y(s) = \frac{k}{(s - j\omega)^m} + \ldots$$



$$\approx \frac{|k|}{\rho^m e^{j\theta . m}} = \underbrace{\left[\frac{|k|}{\rho^m} \cdot (\cos(-\theta . m)\right]}_{\text{real so } \geq 0 \text{ needs } m=1 \text{ also } k \text{ is real}} + j \sin(-\theta . m) \; ; \quad -\pi/2 \leq \theta \leq \pi/2$$

$$y(s) = \frac{k_0}{s} + k_\infty \cdot s + \sum_{\ell=1}^{p} \frac{2k_\ell s}{s^2 + \omega_\ell^2} \qquad \text{all } k\text{'s} \geq 0$$

allows all 4 canonical forms of LPR $y(s)$ to be synthesized with minimum number of L's & C's.

- The Bott-Duffin synthesis allows to synthesize any PR $y(s)$ uses the Richards' function $R(s)$

given $f(s)$ as PR, k a constant

$$R(s) = \left[k f(k) - s f(s)\right] \Big/ \left[k f(s) - s f(k)\right] \qquad \text{is PR if } k \text{ is real \& positive}$$