$2v^i = v + i$

$2v^r = v - i$

$S v^i = v^r$

for all v^i

$$\mathcal{E}(\infty) = \int_{-\infty}^{\infty} v^{iT*}(j\omega)\left[1_n - S^{T*}(j\omega) S(j\omega)\right] v^i(j\omega)\,\frac{d\omega}{2\pi} \geq 0 \text{ if } N \text{ is passive}$$

(if ≤ 0 for some v^i $\Rightarrow$ active)

if $= 0$ N is lossless

Rational BR

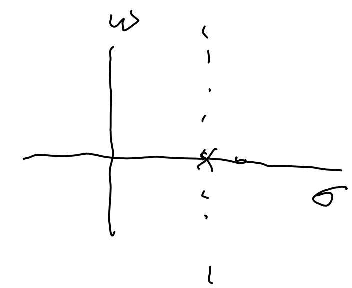
$s = \sigma + j\omega$

1) $S(s)$ has real coefficients

2) $S(s)$ no poles in $\sigma \geq 0$
(analytic in closed RHP)

3) $1_n - S^T(-j\omega) S(j\omega) \geq 0$ positive semidefinite

$\dfrac{1}{s-a}$



$f(t) = e^{at}\,1(t)$

unstable

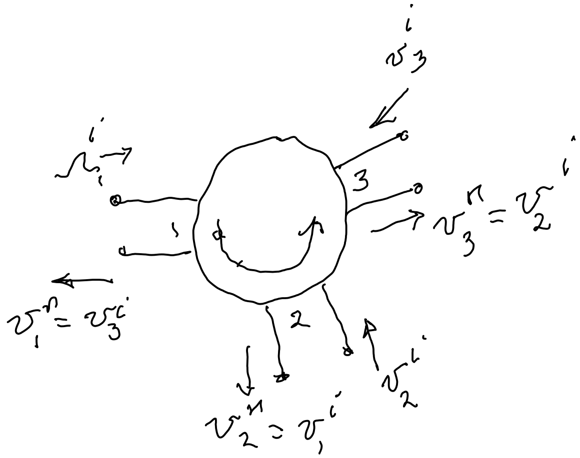
Let $j\omega \Rightarrow s$ $\quad 1_{nn} - S^T(-s)S(s) = 0_n$ if lossless

if lossless $\quad S^T(-s)S(s) = 1_n \Rightarrow S^{-1}(s) = S^T(-s)$

Ex: $\quad S(s)\Big|_{cap} = \dfrac{1-Cs}{1+Cs} = \dfrac{1-y}{1+y} \qquad S(-s)\Big|_{cap} = \dfrac{1-C(-s)}{1+C(-s)} = \dfrac{1+Cs}{1-Cs} = \dfrac{1}{S(s)}$

(checks lossless)

circulator



$v^r = S v^i$

$$\begin{bmatrix} v_1^r \\ v_2^r \\ v_3^r \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} v_1^i \\ v_2^i \\ v_3^i \end{bmatrix}$$

$$S_{cis} = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \Rightarrow 1_3 - \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} = 1_3 - S_{cis}^{T*} S_{cis}$$

$$1_3 - \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{1_3} \equiv 0_3 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Need to look PR = positive real (rational)

def: $Y(s)$

1) $Y(s)$ is real for $\sigma > 0$

2) $Y(s)$ is analytic in $\sigma > 0$

3) $Y^{T*}(s) + Y(s) \geq 0$ in $\sigma > 0$

$$\mathcal{E}(t) = \int_{-\infty}^{t} p(\tau)\,d\tau = \int_{-\infty}^{t} v^T(\tau)\, i(\tau)\,d\tau = \int_{-\infty}^{t} i^T(\tau)\, v(\tau)\,d\tau$$

$$= \int_{-\infty}^{t} \left(\frac{v^T(\tau)\, i(\tau) + i^T(\tau)\, v(\tau)}{2} \right) d\tau$$

$$\int_{-\infty}^{\infty} f^*(t)\, g(t)\,dt = \int_{-\infty}^{\infty} F^*(j\omega)\, G(j\omega)\,\frac{d\omega}{2\pi} \implies$$ Plancherel's theorem if f & g are square integrable

$$I(j\omega) = Y(j\omega)\, V(j\omega) \qquad \mathcal{E}(\infty) = \int_{-\infty}^{\infty} \left(\frac{V^{*T} I + I^{*T} V}{2} \right) \frac{d\omega}{2\pi} \geq 0$$ if passive

$$\mathcal{E}(\infty) = \int_{-\infty}^{\infty} V^{*T} \left(Y(j\omega) + Y^{T*}(j\omega) \right) V(j\omega)\,\frac{d\omega}{2\pi} \implies \frac{Y(j\omega) + Y^{*T}(j\omega)}{2} \geq 0$$

$\dfrac{Y(j\omega) + Y^T(-j\omega)}{2} \geq 0$, set $j\omega = s$ $\Rightarrow$ analytically continue to all s (except poles)

$Y(s) + Y^{T*}(s) \geq 0$ in $\sigma > 0$

$\Rightarrow$ leads to positive real; scalar $y(s) + y^*(s) = 2\,\mathrm{Re}\, y(s)$

also $Y(s) + Y^T(-s) \Rightarrow 2\,\mathrm{Ev}\, y(s) = y(s) + y(-s)$

if lossless $Y(j\omega) + Y^T(-j\omega) = 0$ for almost ω $\Longrightarrow$ scalar $y(s) + y(-s) = 0$ if lossless

$y(s) + y^*(s) \geq 0$ in $\sigma > 0$ (from max

$e^{-y(j\omega)} = e^{-\mathrm{Re}\, y(j\omega)} \cdot e^{-j\,\mathrm{Im}\, y'} \Rightarrow \left| e^{-y} \right|_{s=j\omega} = \left| e^{-\mathrm{Re}\, y(j\omega)} \right|$

$\Downarrow$

y & e^{-y} are analytic in $\sigma > 0$

$\mathrm{Re}\, y(s) \geq 0$ in $\sigma > 0$

$n=1$, $\quad \underbrace{Y(s) + Y(-s)}_{2\,\mathrm{Ev}\,Y(s)} = 0$ if lossless $\Longrightarrow Y(s) = $ odd in $s = \dfrac{s\,N_1}{D_1}$ with N_1 & D_1 even

or $= \dfrac{1}{s}\dfrac{N_2}{D_2}$ with $\underbrace{N_2 \,\&\, D_2}_{\text{polynomial}}$ even

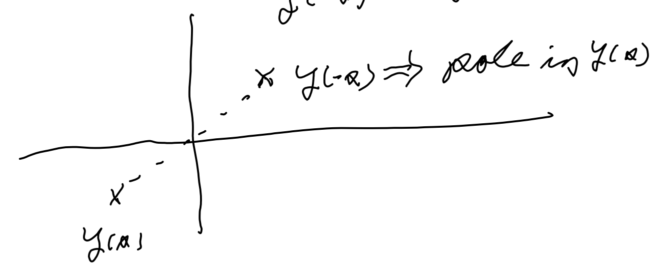
$\therefore$ there is a pole @ $s=0$ or ∞

Ex: $\quad Y(s) = \dfrac{s^5 + 3s^3 + 2s}{s^4 + 5s^2 + 6} = s\left(\dfrac{s^4 + 3s^2 + 2}{s^4 + 5s^2 + 6}\right)$; lossless $Y(s) =$ reactance function

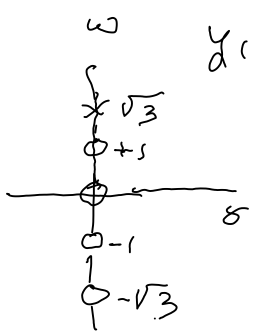
$x = s^2$: $\quad x^2 + 3x + 2 \Longrightarrow x_{1,2} = \frac{-3}{2} \pm \frac{1}{2}\sqrt{9-8} = -1, -2$

$\qquad x^2 + 5x + 6 \Longrightarrow x_{1,2} = \frac{-5}{2} \pm \frac{1}{2}\sqrt{25-24} = -2, -3$

$Y(-s) = -Y(s)$



$Y(s) = \dfrac{s(s^2+1)(s^2+2)}{(s^2+2)(s^2+3)} = \dfrac{s(s^2+1)}{s^2+3} \Longrightarrow$



as O & X do not interleave
this is not an LPR function

no poles in $\sigma < 0$ or $\sigma > 0$

$\Longrightarrow$ all poles are on $j\omega$ axis

$\Longrightarrow$ as $Z(s)$ is PR with $Y(s)$

$\Longrightarrow$ no zeros in $\sigma < 0$, $\sigma > 0$

$\therefore$ all poles & zeroes for a LPR are on the $j\omega$ axis (all poles & zeroes are simple

" lossless

Comments on Spice for scattering matrices: The setup in the Cadence pdf file s-Parameter_ Data_appnote.pdf (on the course web page) shows how to plot the scattering matrix entries for the sinusoidal steady stage for any Sij. Their setup is for 50 Ohm reference resistors so all ports should have 50 Ohm loading. This is already in the excitation port shown in their R2 of Fig. 4 where SRE is the terminal to measure the voltage as the reflection coefficient (diagonal entry ii of S) ; E2 and E1 (of Fig 1) are voltage controlled voltage sources of gain=2 . Figure 1 gives the load circuit to attach to the transmission port (j when j not = i) except a 50 Ohm load is needed from the CKT lead to ground; STR is the terminal at whiich to measure the voltage which is the transmission entry ij, i-not= j, of S. Their Figure 6 gives a nice example of use for obtaining all four entries of S for a 2-port. Note that their incident and reflected voltages have a squre root of Zo=R factor included in Eq. (1) where the denominator Z is a misprint for Zo.