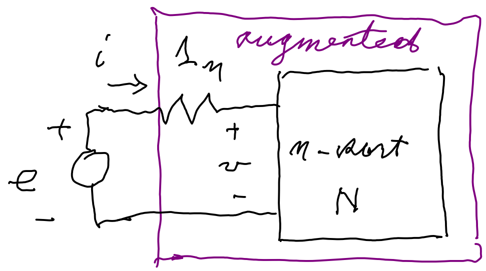


$2v^i = v + i = e$

$2v^r = v - i$

$I = Y_a E$

$V = E - I = E - Y_a E = (1_n - Y_a) E$

$$\int_{-\infty}^{t} e^T(\tau) e(\tau)\, d\tau \geq 0, \quad -\infty \leq t \leq \infty \quad \text{if } e(t) \text{ is square-integrable}$$

$(1_n - Y_a) I = Y_a V$ as $(1_n - Y_a) Y_a E = (1_n - Y_a) I = (Y_a - Y_a^2) E$

$Y_a (1_n - Y_a) E = Y_a V \quad = (Y_a - Y_a^2) E$

$AV = BI$

$A = Y_a, \quad B = 1_n - Y_a$

$V = V^i + V^r$, $I = V^i - V^r$ $\Rightarrow$ $AV = A(V^i + V^r) = B(V^i - V^r)$

$(B+A)V^r = (B-A)V^i$ $\qquad S = (B+A)^{-1}(B-A)$

$S V^i = V^r$

$I = YV \Rightarrow A = Y,\ B = 1_n \Rightarrow S = (1_n + Y)^{-1}(1_n - Y)$ $\Big\}$ 2 ways to get S

also $S = (1_n - Y_a + Y_a)^{-1}(1_n - Y_a - Y_a) = 1_n - 2Y_a$

$s = \sigma + j\omega$

Ex: s-plane

$n=1$

$\underbrace{y = \frac{1}{sL}}$;

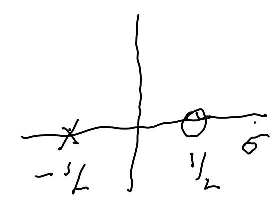
pole at $s=0$

(on $j\omega$ axis)

$S_L = \left(1 + \frac{1}{sL}\right)^{-1}\left(1 - \frac{1}{sL}\right) = \frac{sL}{1+sL} \times \frac{sL-1}{sL} = \frac{sL-1}{sL+1}$

no pole on $s = j\omega$ $\quad$ pole is @ $s = -\frac{1}{L}$

zero @ $s = \frac{1}{L}$



If N is passive $S(s)$ exists & has no poles on $j\omega$ axis
& is bounded real = BR if rational

Bounded real definition, $S(s)$, $s = \sigma + j\omega$

1) $S(\sigma)$ is real if $\sigma > 0$ — real circuits
2) $S(s)$ is analytic in $\sigma > 0$ — stability
3) $1_n - \|S(s)\|^2 \geq 0$ in $\sigma > 0$ — passivity

For 3): Let $e(t) \in L^2$, $e = v + i$

$$\underbrace{\int_{-\infty}^{\infty} e^T(\tau) e(\tau)\, d\tau}_{\substack{\geq 0 \\ \text{(finite)}}} = \int_{-\infty}^{\infty} (v+i)^T (v+i)\, d\tau = \int_{-\infty}^{\infty} \Big[\underbrace{\underbrace{v^T(\tau) v(\tau)}_{\geq 0} + \underbrace{i^T(\tau) i(\tau)}_{\geq 0}}_{\text{as sums of squares}} + \underbrace{2 v^T(\tau) i(\tau)}_{\geq 0 \text{ if passive}}\Big] d\tau$$

$\Longrightarrow$ all terms finite if passive

as $\int_{-\infty}^{\infty} p(\tau)\, d\tau \geq 0$

∴ if passive $\mathcal{F}[e] = \int_{-\infty}^{\infty} e(\tau) e^{-j\omega\tau} \frac{d\,2\pi f}{2\pi}$ exists if $e \in L^2$

Parseval's

$$\int_{-\infty}^{\infty} e(t)^{T*} e(t)\, dt = \int_{-\infty}^{\infty} \mathcal{F}[e(t)]^{T*} \mathcal{F}[e(t)] \frac{d\omega}{2\pi}$$

$$= \int_{-\infty}^{\infty} E^{T*}(j\omega)\, E(j\omega) \frac{d\omega}{2\pi} \qquad \int_{-\infty}^{\infty} V^{rT*}(j\omega)\, V^{r}(j\omega) \frac{d\omega}{2\pi} = \int V^{iT*}(j\omega) S^{T*}(j\omega) S(j\omega) V^{i}(j\omega)\, d\omega$$

$$\underbrace{\int_{-\infty}^{\infty} V^{iT*}(j\omega) V^{i}(j\omega) \frac{d\omega}{2\pi}}_{\text{incident}} - \underbrace{\int_{-\infty}^{\infty} V^{iT*}(j\omega) S^{T*}(j\omega) S(j\omega) V^{i}(j\omega) \frac{d\omega}{2\pi}}_{\text{reflected}} \geq 0$$

by passivity

$$\int_{-\infty}^{\infty} V^{iT*}\left[1_n - S^{T*}(j\omega)S(j\omega)\right]V^{i}(j\omega)\,\frac{d\omega}{2\pi} \geq 0 \quad \text{for all } V^{i}(j\omega)$$

$$\Longrightarrow \quad 1_n - S^{T*}(j\omega)S(j\omega) \text{ is a positive semi-definite matrix}$$

if N is passive.

no poles on $j\omega$ axis

Lossless if ≥ 0 becomes $0 \Rightarrow 1_n = S^{T*}(j\omega)S(j\omega)$

Ex: $S(s) = \frac{Ls-1}{Ls+1}$; $s = j\omega$, $S^{T*}(j\omega) = \left(\frac{L(j\omega)-1}{L(j\omega)+1}\right)^* = \frac{-Lj\omega-1}{L(-j\omega+1)} \Rightarrow \frac{1}{S(j\omega)} = \frac{Lj\omega-1}{Lj\omega+1}$

$$= \frac{Lj\omega+1}{Lj\omega-1}$$

$$\times\, S(j\omega) = \frac{Lj\omega-1}{Lj\omega+1} \Rightarrow S^{T*}(j\omega)S(j\omega) = 1 \Rightarrow$$ inductor is lossless

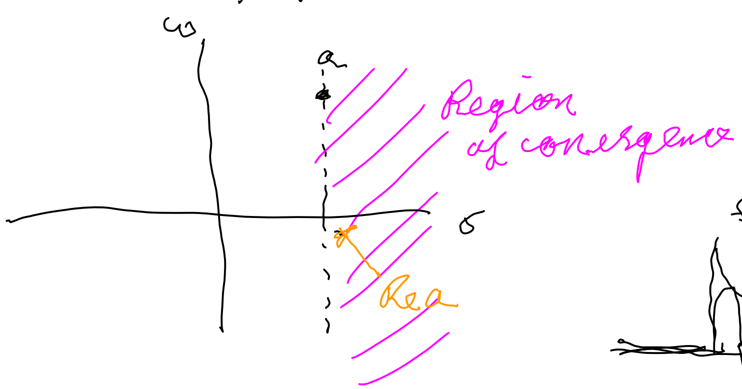
Bilateral Laplace transform

$$\mathcal{L}[f(t)] = \int_{-\infty}^{\infty} f(t)\, e^{-st}\, dt \quad , \quad s = \sigma + j\omega \quad ; \qquad 1(t) = \begin{cases} 0 & t < 0 \\ 1 & t > 0 \end{cases}$$

Ex: $\quad f(t) = e^{at}\, 1(t)$

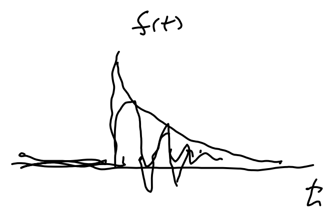
$$\mathcal{L}[t] = F(s) = \int_{-\infty}^{\infty} e^{at} e^{-st}\, 1(t)\, dt = \int_{-\infty}^{\infty} e^{(a-s)t}\, 1(t)\, dt$$

s-plane $\quad s = \sigma + j\omega$



$$= \int_{0}^{\infty} e^{(a-s)t}\, dt = \frac{1}{a-s} \cdot e^{(a-s)t} \Big|_{0}^{t=\infty} = \begin{cases} 0 & \text{if } \operatorname{Re} s > \operatorname{Re} a \\ \frac{-1}{a-s} = \frac{1}{s-a} \end{cases}$$

does not exist if $\operatorname{Re} s < \operatorname{Re} a$

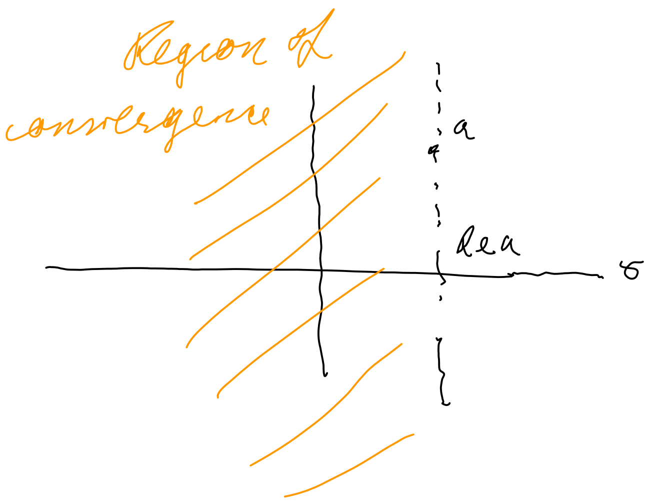


$\mathcal{L}\left[-e^{at}\,1(-t)\right]$

$$F(s) = \int_{-\infty}^{\infty} -e^{(a-s)t}\,1(-t)\,dt = \int_{-\infty}^{0} -e^{(a-s)t}\,dt$$

$$= \frac{-1}{a-s}\,e^{(a-s)t}\Big|_{-\infty=t}^{0} = \frac{-1}{a-s} \quad \text{if } \operatorname{Re}(a-s) \geq 0$$

$$= \frac{1}{s-a}$$



$\therefore$ for a passive rational $S(s)$ it is bounded real ($\Rightarrow$ BR) and region of convergence includes $\sigma \geq 0$