ENEE 610
09/19/19

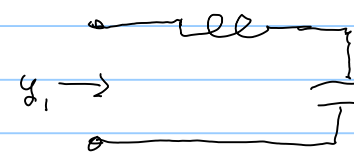
Note Title 9/18/2019

$$y_1 = \left(\frac{G}{2}\right)\left(\frac{1-A_v}{1+A_v}\right) \qquad A_v(s) = k\left(\frac{s^2 - as + b}{s^2 + as + b}\right) \qquad \begin{array}{l} a, b \text{ real} \\ k \text{ real} \end{array}$$

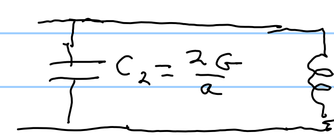
$$y_1 = \frac{G}{2}\left(\frac{s^2 + as + b - k(s^2 - as + b)}{s^2 + as + b + k(s^2 - as + b)}\right) = \frac{G}{2}\left(\frac{(1-k)s^2 + (1+k)as + (1-k)b}{(1+k)s^2 + (1-k)as + (1+k)b}\right)$$

choose $k = 1 \implies y_1 = \frac{G}{2}\left[\frac{2as}{2s^2 + 2b}\right] = \frac{G}{2} \cdot \frac{as}{s^2 + b} = \frac{G}{2}\,\frac{1}{\frac{s}{a} + \frac{b}{as}}$

$$z_1 = \frac{1}{y_1} = \frac{2}{G}\frac{s}{a} + \frac{2}{G}\frac{b}{as} = Ls + \frac{1}{Cs}$$

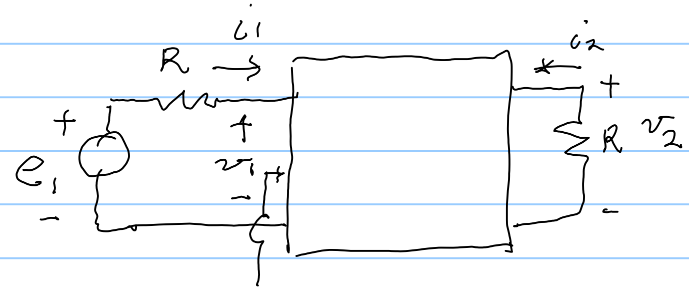


$$y_2 = \frac{G^2}{y_1} = G^2 z_1 = \frac{2G}{a}s + \frac{2Gb}{as}$$



duals

Choose $k = -1 \Rightarrow y_1 = \frac{G}{2}\left(\frac{-2\alpha^2 - 2b}{-2a\alpha}\right) = \frac{G}{2} \cdot \frac{\alpha^2 + b}{a\alpha}$



$R = Z_{in}$

$G = 1/R = y_{in}$

$\frac{v_2}{v_1} = A_v$

$e_1 = R \cdot i_1 + R\, i_{in} = 2 v_1 \Rightarrow \frac{v_2}{e_1} = \frac{v_2}{2v_1} = \frac{1}{2} A_v$

$i_{in} = i_1$

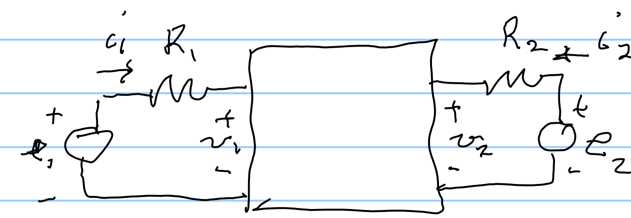
$2v^i = v + Ri$

$2v^r = v - Ri$

v^i = incident voltage

v^r = reflected voltage

$v = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}, \quad i = \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$



$R = \begin{bmatrix} R_1 & 0 \\ 0 & R_2 \end{bmatrix}$

$$V^r = S V^i \quad , \quad S = \begin{bmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \end{bmatrix}$$

& if $e_2 = 0$, $e_1 = v_1 + R_1 i_1 = 2 v_1^i$

$v_2 = -R_2 i_2$

$\Rightarrow v_2^i = 0$, $2v_2^r = v_2 - R_2 i_2 = 2 v_2$

$$\begin{bmatrix} v_1^r \\ v_2^r \end{bmatrix} = \begin{bmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \end{bmatrix} \begin{bmatrix} v_1^i \\ \underset{\substack{\shortparallel \\ 0}}{v_2^i} \end{bmatrix} \Longrightarrow v_2^r = \alpha_{21} \cdot v_1^i \Rightarrow \frac{v_2^r}{v_1^i} = \alpha_{21} = \frac{v_2^r}{e_1/2} \Rightarrow \frac{v_2^r}{e_1} = \frac{\alpha_{21}}{2}$$

$$v_2^r = 2 v_2 \qquad \frac{v_2^r}{v_1^i} = \alpha_{21} = \frac{2 v_2}{v_1^i} \Rightarrow \frac{v_2}{v_1^i} = \frac{1}{2} \alpha_{21}$$

$$\text{Energy}(t) = \int_{-\infty}^{t} p(\tau)\, d\tau = \int_{-\infty}^{t} v^T(\tau)\, i(\tau)\, d\tau \quad , \quad t > -\infty \text{ and as } t \to \infty$$

$$= \int_{-\infty}^{t} \left(\frac{v^T(\tau) i(\tau) + i^T(\tau) v(\tau)}{2} \right) d\tau$$

$$2v^i = v + Ri \qquad 2v = 2v^i + 2v^r \implies v = v^i + v^r$$
$$2v^r = v - Ri \qquad 2Ri = 2v^i - 2v^r \qquad i = R^{-1}(v^i - v^r) = G(v^i - v^r)$$

$R^{-1} = G$ matrices

$G = G^T$

$$v^T i = (v^i + v^r)^T G (v^i - v^r)$$
$$= v^{iT} G v^i - v^{iT} G v^r + \underbrace{v^{rT} G v^i}_{(v^{iT} G^T v^r)^T} - v^{rT} G v^r$$

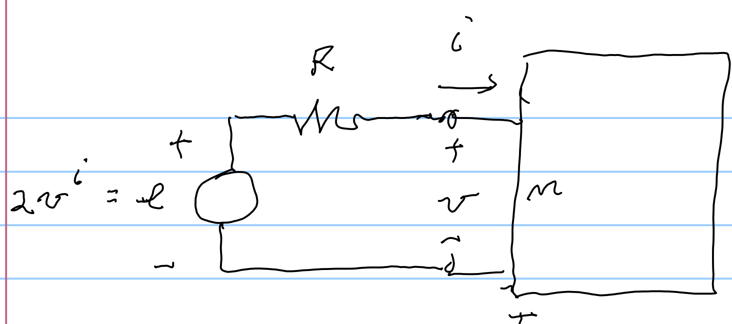
$$v^T i = p(t) = v^{iT} G v^i - v^{rT} G v^r$$

if normalize $G = I_n$ for n-ports

$$p(t) = v^{iT} v^i - v^{rT} v^r$$

$$\text{Energy}(t) = \int_{-\infty}^{t} \Big[\underbrace{v^{iT}(\tau) v^i(\tau)}_{\text{incident power}} - \underbrace{v^{rT}(\tau) v^r(\tau)}_{\text{reflected power}} \Big] d\tau$$

if ≥ 0 call the n-port is passive



assume $e^T e$ is "integrable"

$$e = v + Ri = 2v^i$$
$$v - Ri = 2v^r$$

$$e^T e = (v+Ri)^T(v+Ri) = v^Tv + i^TR^Tv + v^TRi + i^TR^TRi = \frac{1}{4} v^{iT} v^i$$
$$\underbrace{v^{rT} v^r}_{2\times 2} = (v-Ri)^T(v-Ri) = v^Tv - i^TR^Tv - v^TRi + i^TR^TRi = \frac{1}{4} v^{rT} v^r$$

$$\frac{1}{4}\left[v^{iT}v^i - v^{rT}v^r\right] = 2 \cdot i^Tv + 2v^Ti$$

$R = I_n$ normalization

if passive $\displaystyle\int_{-\infty}^{t} \frac{(i^Tv + v^Ti)}{2}\, d\tau \geq 0$ for all t

$$= \frac{1}{16}\int_{-\infty}^{t} \left[v^{iT}v^i - v^{rT}v^r\right] \geq 0$$

if v^i is "square integrable" then so is v^r if passive (for all $t \to \infty$)

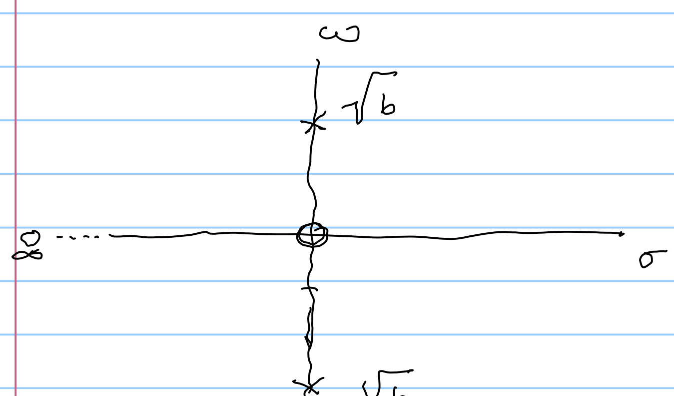
Parseval's theorem

$$\int_{-\infty}^{\infty} f(t)^{*} f(t)\, dt = \int_{-\infty}^{\infty} \mathcal{F}[f]^{*}\, \mathcal{F}[f] \cdot \frac{d\omega}{2\pi} \quad , \quad \omega = 2\pi f$$

(frequency)

function

here $s = \sigma + j\omega \Rightarrow j\omega$



$$\frac{G}{2} \cdot \frac{as}{s^2 + b} \qquad s = d/dt \, , \quad s = \sigma + j\omega$$

zeros @ $s = 0, \infty$

poles @ $s = \pm\sqrt{-b} = \pm j\sqrt{b}$

$$y_1 = \frac{G}{2} \cdot \frac{as}{s^2 + b} \, ,$$

$i = y v$, $i = \frac{G}{2} \frac{a s}{s^2 + b} v \Rightarrow (s^2 + b) i = \frac{G}{2} a s v \Rightarrow \frac{d^2 i}{dt^2} + b i = \frac{G}{2} a \frac{dv}{dt}$

using the Laplace transform $\mathcal{L}[f] = \int_{-\infty}^{\infty} f(t) e^{-st} \, dt \Rightarrow$ bilateral Laplace transform

$s = \sigma + j\omega$