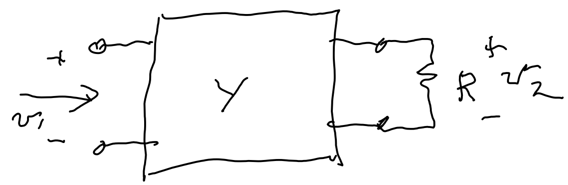
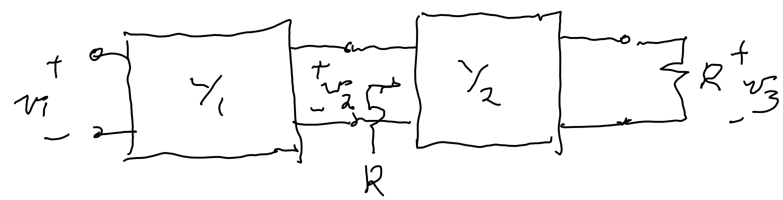
ENEE 610
09/17/19

Note Title 9/17/2019

Constant R

$z_{in} = R$

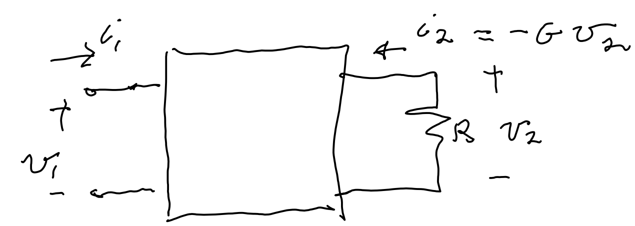




$\frac{v_3}{v_1} = \frac{v_3}{v_2} \cdot \frac{v_2}{v_1} \Rightarrow$ multiply voltage gains if constant R

y_{in} given $Y_{2\text{-port}} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix}$ load in $R = 1/G$

$y_{in} = G$



$y_{in} = i_1 / v_1$ $\qquad i_1 = y_{11} v_1 + y_{12} v_2$

$-G v_2 = i_2 = y_{21} v_1 + y_{22} v_2 \Rightarrow (-G - y_{22}) v_2 = y_{21} v_1$

$i_1 = y_{11} v_1 + y_{12} (-G - y_{22})^{-1} y_{21} v_1 \Rightarrow y_{in} = y_{11} - y_{12} \dfrac{1}{G + y_{22}} \cdot y_{21}$

if $y_{in} = G \Rightarrow G = y_{in} = \dfrac{G y_{11} + y_{11} y_{22} - y_{12} y_{21}}{G + y_{22}} = \dfrac{G y_{11} + \Delta y}{G + y_{22}}$, $\Delta y =$ determinant of Y

$G^2 + G y_{22} = G y_{11} + \Delta y$

$G^2 + G(y_{22} - y_{11}) = y_{22} y_{11} - y_{12} y_{21}$ $\qquad$ choose $y_{11} = y_{22} = y_1 + y_2$, $y_{12} = y_{21} = y_1 - y_2$

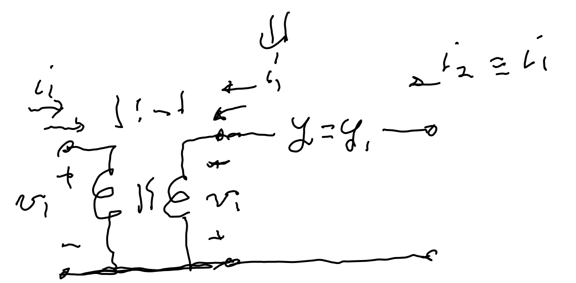
becomes $G^2 + G(0) = (y_1 + y_2)(y_1 + y_2) - (y_1 - y_2)(y_1 - y_2)$

$= y_1^2 + y_2^2 + 2 y_1 y_2 - [y_1^2 + y_2^2 - 2 y_1 y_2] = 4 y_1 y_2 \Rightarrow y_1 y_2 = G^2/4 = (G/2)^2$

$$Y = \begin{bmatrix} y_1 + y_2 & y_1 - y_2 \\ y_1 - y_2 & y_1 + y_2 \end{bmatrix} = \begin{bmatrix} y_1 & y_1 \\ y_1 & y_1 \end{bmatrix} + \begin{bmatrix} y_2 & -y_2 \\ -y_2 & y_2 \end{bmatrix} \quad , \quad y_1 = \frac{G^2}{4y_2} \text{ or } y_2 = G^2/4y_1$$

$$= \begin{bmatrix} y_1 & y_1 \\ y_1 & y_1 \end{bmatrix} + \begin{bmatrix} G^2/4y_1 & -G^2/4y_1 \\ -G^2/4y_1 & G^2/4y_1 \end{bmatrix}$$

$\Downarrow$



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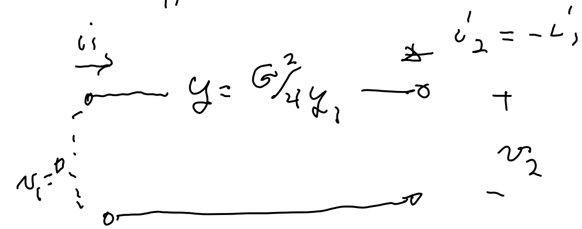
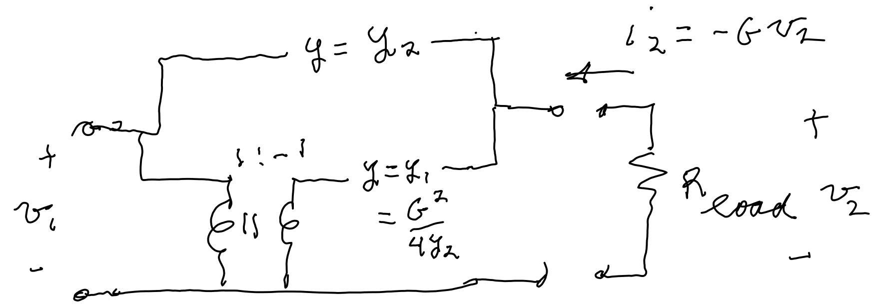
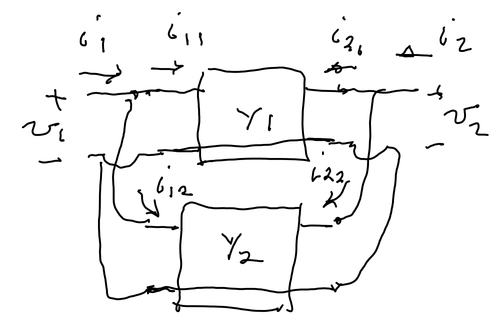




$y_{in} = G$ when $R_{load} = 1/G$



$Y = Y_1 + Y_2$

need v_2/v_1 $\qquad i_2 = y_{21} v_1 + y_{22} v_2 = -G v_2$

$$\Rightarrow \quad y_{21} v_1 = (-G - y_{22}) v_2 \Rightarrow \frac{v_2}{v_1} = \frac{-y_{21}}{y_{22} + G} = \frac{-(y_1 - y_2)}{(y_1 + y_2 + G)}$$

$$A_v = \frac{v_2}{v_1} = \frac{-\left(y_1 - \frac{G^2}{4y_1}\right)}{\left(y_1 + \frac{G^2}{4y_1} + G\right)} = \frac{G^2 - 4y_1^2}{G^2 + 4y_1 G + 4y_1^2} = \frac{(G - 2y_1)(G + 2y_1)}{4\left[(G/2)^2 + 2 G/2 \cdot y_1 + y_1^2\right]}$$

$$= \frac{(G - 2y_1)(G + 2y_1)}{4(G/2 + y_1)^2} = \frac{2(G - 2y_1)(G/2 + y_1)}{4(G/2 + y_1)^2} = \frac{4(G/2 - y_1)}{4(G/2 + y_1)} = \frac{G/2 - y_1}{G/2 + y_1}$$

if $y_1 = \frac{1}{sL}$ an inductor: $\quad A_v = \frac{\left(G/2 - \frac{1}{sL}\right)}{G/2 + 1/sL} = \frac{s - \frac{2}{GL}}{s + 2/GL} = \frac{v_2}{v_1}(s), \quad s = \sigma + j\omega$

this is all-pass, $s = j\omega$

$$A_v(j\omega) = \frac{j\omega - 2/GL}{j\omega + 2/GL}; \quad A_v(j\omega) = \frac{(j\omega - 2/GL)(-j\omega + 2/GL)}{(j\omega + 2/GL)(-j\omega + 2/GL)} = \frac{\left(\omega^2 - 4/G^2L^2\right) + j\omega\left(\frac{4}{GL}\right)}{\omega^2 + (4/GL)^2}$$

$$|A_v(j\omega)| = \frac{|j\omega - 2/GL|}{|j\omega + 2/GL|} = \frac{\sqrt{\omega^2 + (2/GL)^2}}{\sqrt{\omega^2 + (2/GL)^2}} = 1 \Rightarrow$$ the all pass changes phase of sinusoids but not amplitudes

also given A_v can find $y \Rightarrow \quad A_v\left(\frac{G}{2}+y_1\right) = G/2 - y_1$

$$(A_v+1)\,y_1 = G/2 - G/2\,A_v = G/2\,(1-A_v) \Rightarrow y_1 = \left(\frac{G}{2}\right)\left(\frac{1-A_v}{1+A_v}\right)$$

also if $y_1(\alpha)$ is rational, $y_1 = \frac{n_1}{d_1} \Rightarrow A_v = \dfrac{G/2 - n_1/d_1}{G/2 + n_1/d_1} = \dfrac{d_1(G/2) - n_1}{d_1(G/2)+n}$

$n_1 = \text{odd}(\alpha) \qquad d_1 = \text{even}(\alpha)$

Ex: $\quad P(\alpha) = 4\alpha^5 + 3\alpha^4 + 2\alpha^3 - 4\alpha^2 - 2\alpha + 8 = \underbrace{(3\alpha^4 - 4\alpha^2 + 8)}_{\text{Ev}\,P(\alpha)} + \underbrace{\alpha(4\alpha^4 + 2\alpha^2 - 2)}_{\text{Od}\,P(\alpha)}$

$\alpha = j\omega$; Even(α) = real, odd = imaginary

if $y_1 = \dfrac{\text{Od}(\alpha)}{\text{Ev}(\alpha)} = \dfrac{n_1}{d_1}$ then A_v is all-pass $\Leftrightarrow |A_v(j\omega)| = 1$

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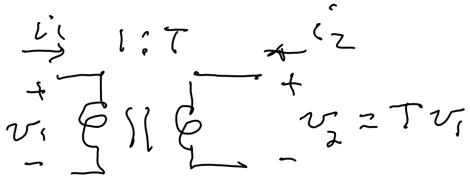
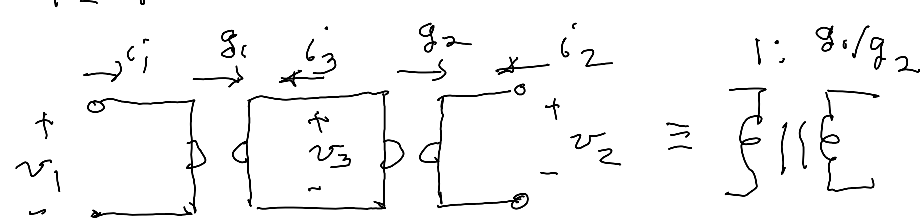


$v_1 i + v_2 i_2 \equiv 0$

$= \cancel{v_1} i_1 + \cancel{v_1} T i_2 \Rightarrow i_1 = -T i_2$

above $T = -1$

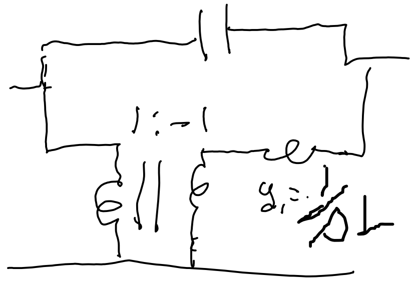


$i_1 = g_1 v_3 \qquad -i_3 = g_2 v_2 \qquad \Rightarrow -(-g_1 v_1) = g_2 v_2 \Rightarrow v_2 = \frac{g_1}{g_2} v_1$

$i_3 = -g_1 v_1 \Longleftarrow \qquad i_2 = -g_2 v_3 \qquad \Rightarrow i_2 = -g_2 (i_1/g_1)$

$i_1 = -\frac{g_1}{g_2} i_2$

$y_2 = \frac{G^3}{4 y_1} = \frac{G^2}{4} \cdot sL \qquad = -g_2/g_1$



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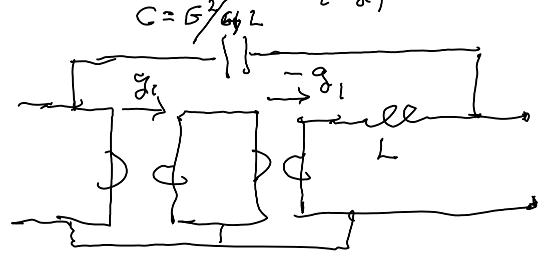
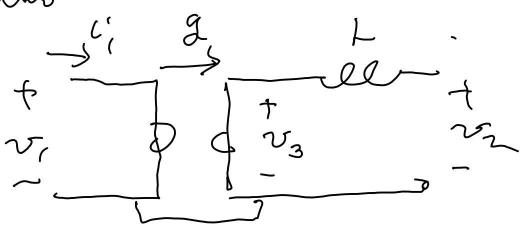


also



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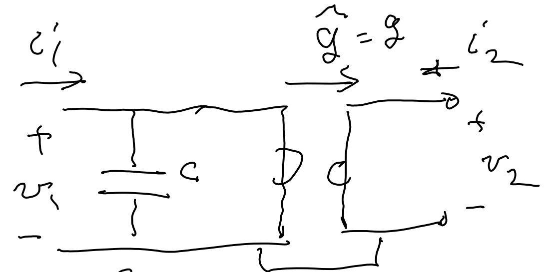
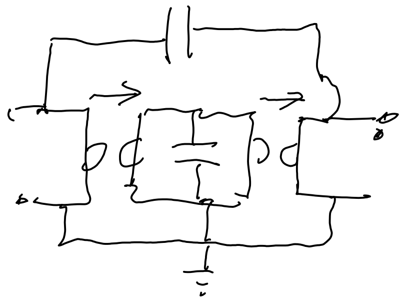


$C = g^2 L$

$i_2 = -g v_1 \qquad i_1 = sC v_1 + \hat{g} v_2 \qquad \hat{g} = g$

$i_1 = g v_3 \qquad sC = g^2 sL$

$(sL)(i_2) = v_2 - v_3 = v_2 - i_1/g \Longrightarrow \quad i_1 = g v_2 - g sL\, i_2 = g v_2 - g sL(-g v_1)$



$$A_v = \frac{\alpha - 2/G_L}{\alpha + 2/G_L}$$