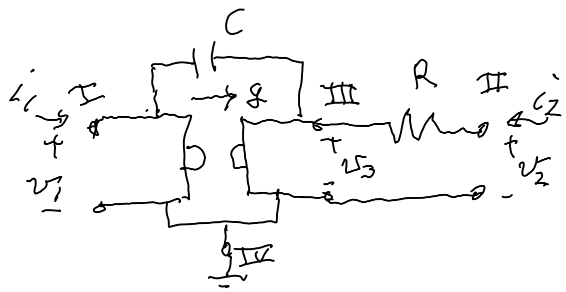
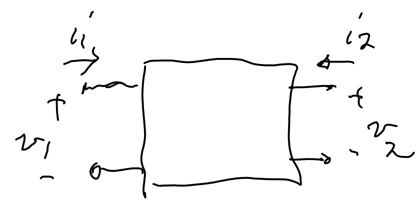
Note Title 9/8/2019





$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = Z \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$$

$$\begin{bmatrix} i_1 \\ i_2 \\ 0 = i_3 \end{bmatrix} = \begin{bmatrix} Cs & 0 & g - Cs \\ 0 & G & -G \\ -g - Cs & -G & G + Cs \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} \Rightarrow s \begin{bmatrix} C & 0 & -C \\ 0 & 0 & 0 \\ -C & 0 & C \end{bmatrix} v = \begin{bmatrix} 0 & 0 & -g \\ 0 & -G & G \\ g & G & -G \end{bmatrix} v + \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$$

set $i_3 = 0$

$$sEx \doteq Ax + Bu, \quad x = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}, \quad u = \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$$

$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$$

y = output
u = input

$$y = Cx$$

$y = Cx$, $\quad (\alpha E - \mathcal{A})x = \mathcal{B}u$; $\quad x = (\alpha E - \mathcal{A})^{-1}\mathcal{B}u$

$= C(\alpha E - \mathcal{A})^{-1}\mathcal{B}u$ here $Z = C(\alpha E - \mathcal{A})^{-1}\mathcal{B}$

$\uparrow$ output $\qquad \uparrow$ input

$$Z(\alpha) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \alpha C & 0 & g - \alpha C \\ 0 & G & -G \\ -g - \alpha C & -G & \alpha C + G \end{bmatrix}^{-1} \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$$

to get the inverse : det $= \alpha CG(\alpha C + G) + 0 + 0 - [-(g+\alpha C)(g - \alpha C)G + G^2 \alpha C]$

$= \alpha^3 C^2 G + \alpha g G^2 + g^2 G - \alpha^2 C^3 G - G^2 \alpha C = g^2 G = \det$

$\Delta_{11} = G(\alpha C + G) - G^2 = \alpha CG$ $\qquad \Delta_{21} = (-1)(Gg - \alpha CG)$ $\qquad \Delta_{31} = -Gg + G\alpha C$

$\Delta_{13} = (-1)^3(-G(g + \alpha C)$ $\qquad \Delta_{22} = \alpha C(\alpha C + G) + (g^2 - (\alpha C)^2)$ $\qquad \Delta_{32} = (-1)(-G\alpha C)$

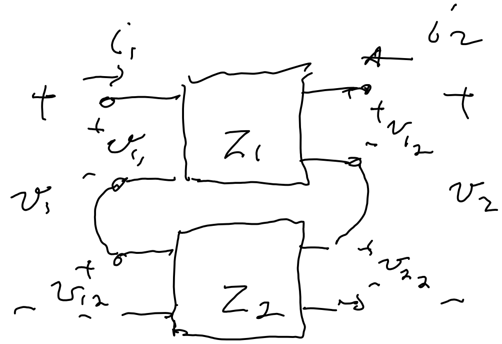
$\Delta_{13} = gG + \alpha CG$ $\qquad \Delta_{23} = (-1)(-G\alpha C)$ $\qquad \Delta_{33} = \alpha CG$

$$Z(\alpha) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \frac{1}{g^2 G} \begin{bmatrix} \alpha C G & -Gg + \alpha C G & -G(g-\alpha C) \\ G(g+\alpha C) & g^2 + \alpha C G & G\alpha C \\ G(g+\alpha C) & \alpha C G & \alpha C G \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$$

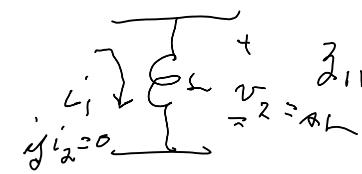
$$= \frac{1}{g^2 G} \begin{bmatrix} \alpha C G & -Gg + \alpha C \cdot G & -G(g-\alpha C \\ G(g+\alpha G) & g^2 + \alpha C G & G\alpha C \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$$

$$= \frac{1}{g^2 G} \begin{bmatrix} \alpha C G & -Gg + \alpha C G \\ Gg + \alpha C G & g^2 + \alpha C G \end{bmatrix} = \alpha \underbrace{\begin{bmatrix} C/g^2 & C/g^2 \\ C/g^2 & C/g^2 \end{bmatrix}}_{Z_\alpha} + \underbrace{\begin{bmatrix} 0 & -1/g \\ 1/g & 0 \end{bmatrix}}_{Z_{gyr}} + \underbrace{\begin{bmatrix} 0 & 0 \\ 0 & 1/G \end{bmatrix}}_{Z_R}$$

$$L = C/g^2$$



$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}_1 + \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}_2 = Z_1 i + Z_2 \cdot i = [Z_1 + Z_2]\, i = Z_{series} \cdot i$$

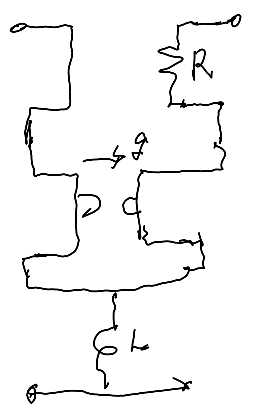


$z_{11} = z_{22} = sL$

$R = 1/G$

for our case

$z_{12} = z_{21} = \left.\dfrac{v_2}{i_1}\right|_{i_2 = 0}$



$Z = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix}$

$v_1 = z_{11} i_1 + z_{12} i_2$

$v_2 = z_{21} i_1 + z_{22} i_2$

$sL \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

an equivalent circuit

$v_2 = z_{21} i_1$ if $i_2 = 0$

$$1_2 v = Z i$$

$$Y v = 1_2 i$$

$$A(s) \cdot v = B(s) i$$

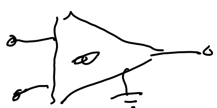
now v & i are port variables

$$1(t) = \begin{cases} 1 & \text{if } t > 0 \\ 0 & \text{if } t < 0 \end{cases}$$

= unit step function

1_n = $n \times n$ identity

$$= \begin{bmatrix} 1 & 0 & 0 & \cdots \\ 0 & 1 & 0 & \cdots \\ \vdots & & \ddots & 0 \\ 0 & \cdots & 0 & 1 \end{bmatrix}$$



$\Rightarrow$ $v_1 = 0$

$i_1 = 0$

v_2, i_2 don't

$$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$$

$$Z = A^{-1} \cdot B$$

$$Y = B^{-1} A$$

$2v^i = v + Ri$ $\quad$ v^i = incident

$2v^r = v - Ri$ $\quad$ v^r = reflected

$v^r = S v^i$, $\quad$ S = scattering matrix

$Zv = 2(v^i + v^r)$ $\Rightarrow$ $A(v^i + v^r) = B \cdot R^{-1}(v^i - v^r)$

$2Ri = 2(v^i - v^r)$ $\quad$ $(A - BR^{-1})v^i = (BR^{-1} + A)(-v^r)$ $\quad$ $v^r = (BR^{-1} + A)^{-1}(BR - A) \cdot v^i$

if $R = 1_n$, A & B $n \times n$ $\quad$ $S = (B+A)^{-1}(B-A)$ $\quad$ if Z then $(A(A^{-1}B + 1_n))^{-1}(A(A^{-1}B - 1_n))$

$$S = (Z + 1_n)^{-1} A^{-1} A (Z - 1_n)$$

$$= (Z + 1_n)^{-1}(Z - 1_n)$$

if Y exists $= (1_n + B^{-1}A)^{-1}(1_n - B^{-1}A) = (1_n + Y)^{-1}(1_n - Y)$