

$$Y = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & -g & 0 & 0 \\ 0 & 0 & g & 0 \\ 0 & 0 & 0 & g \end{bmatrix}$$

$g = (C - g) - g$

Yind  $\Rightarrow$  sum of rows = 0  
 " " " " " columns = 0



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for the capacitor

$$Y_C = \begin{bmatrix} CA & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -CA & 0 & CA & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

I      II      III      IV

$$Y_R = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & G & -G & 0 \\ 0 & -G & G & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

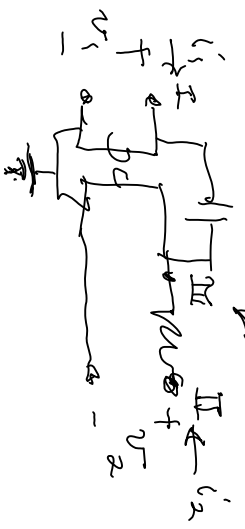
I      II      III      IV

$$Y_{ind} = \begin{bmatrix} C_A & 0 & g-C_A & -g \\ 0 & G & -G & 0 \\ -g-C_A & 0 & G & g \\ g & 0 & -g & 0 \end{bmatrix}$$

*can ignore*

$$\begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} C_A & 0 & g-C_A \\ 0 & G & -G \\ -g-C_A & -G & G+C_A \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$$

if derive 2-port Y for



external current  $i_3 = 0$   
 allows  $v_3$  as a function  
 of  $v_1$  &  $v_2$

more ground to node II

$E_4 = 0$  (also current into node is  
 The sum of all others so  
 can ignore

*multiplies by 0*

$$Y_{external} = \begin{bmatrix} C_A & 0 & g-C_A \\ 0 & G & -G \\ -g-C_A & -G & G+C_A \end{bmatrix}$$

$$\begin{bmatrix} i_1 \\ i_2 \\ 0 \end{bmatrix} = \begin{bmatrix} c_A & 0 & g-c_A \\ 0 & \gamma_{11} & -g \\ -g-c_A & -g & g+c_A \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$$

$$0 = \gamma_{21} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} + \gamma_{22} v_3$$

$$\Rightarrow v_3 = -\gamma_{22}^{-1} \gamma_{21} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

$$Y_{2 \times 2} = \begin{bmatrix} c_A & 0 \\ 0 & g \end{bmatrix} - \begin{bmatrix} g-c_A \\ -g \end{bmatrix} \frac{1}{g+c_A} \begin{bmatrix} -g+c_A \\ -g \end{bmatrix}$$

$$\Rightarrow 0 = [-g-c_A \quad -g] \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} + [g+c_A] \cdot v_3$$

$$\Rightarrow v_3 = \frac{1}{g+c_A} [-g-c_A \quad -g] \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \gamma_{11} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} - \gamma_{12} \gamma_{22}^{-1} \gamma_{21} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

$$= \left\{ \gamma_{11} - \gamma_{12} \gamma_{22}^{-1} \gamma_{21} \right\} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = Y_{2 \times 2} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

$$\begin{aligned}
 \frac{1}{2} \text{rank} &= \begin{bmatrix} c & a & 0 \\ 0 & g & \dots \end{bmatrix}^{-1} \frac{1}{a+c} \begin{bmatrix} -(g^2 - ca^2) & -Gg + Gc a \\ Gg + Gc a & G^2 \end{bmatrix} = \frac{1}{a+c} \begin{bmatrix} Gc a + g^2 & Gg - Gc a \\ -Gg - Gc a & Gc a \end{bmatrix} \\
 &= \frac{1/c}{a + g/c} \begin{bmatrix} Gc(a + g^2/c) & -Gc(a - g/c) \\ -Gc(a + g/c) & Gc a \end{bmatrix}
 \end{aligned}$$

Now set up state eqn.

$$\frac{1}{2} \text{rank} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} \Rightarrow \begin{bmatrix} Gc a + g^2 & -Gc a + Gg \\ -Gc a - Gg & Gc a \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = (c(a+g)) \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$$

$$A \begin{bmatrix} G_C \\ -G_C \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -g_1 & -g_2 \\ g_1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + (cr+e) \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$$

Before  $AEu = Ax + Bu$   $u = \text{input}$ ,  $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

can transform  $x = T_n^{-1} \hat{x}$ ,  $T_R^{-1}$  existing

$$A E T_n^{-1} \hat{x} = A T_n^{-1} \hat{x} + B u$$

& multiply on left by  $T_R$ ,  $T_R^{-1}$  existing

$$A T_R E T_n^{-1} \hat{x} = T_R A T_n^{-1} \hat{x} + T_R B u \Rightarrow A E \hat{x} = A \hat{x} + B u$$

(identical circuit but new  
descriptions)

on this example choose  $T_n$  to add 1st column to column

$$\begin{bmatrix} GC & -GC \\ -GC & GC \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} GC & 0 \\ -GC & 0 \end{bmatrix} \Rightarrow T_n = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

$$T_n \vec{x} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} GC & 0 \\ -GC & 0 \end{bmatrix} = \begin{bmatrix} GC & 0 \\ 0 & 0 \end{bmatrix} \quad \vec{x} = T_n^{-1} \vec{y} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} y_1 - y_2 \\ y_2 \end{bmatrix}$$

$$A T_n \vec{y} = \begin{bmatrix} GC & 0 \\ 0 & 0 \end{bmatrix} \quad T_n = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} -g^2 & -GC \\ GC & 0 \end{bmatrix} T_n = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} -g^2 & -GC \\ GC & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} -g^2 & -g^2 - GC \\ -g^2 + GC & -g^2 \end{bmatrix} = \begin{bmatrix} -g^2 & -GC \\ -g^2 + GC & -g^2 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

$$\begin{aligned}
 & \begin{bmatrix} Gc & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} v_1 - v_2 \\ v_2 \end{bmatrix} = \begin{bmatrix} -g^2 & -Gc \\ -g^2 + Gc & -Gc \end{bmatrix} \begin{bmatrix} v_1 - v_2 \\ v_2 \end{bmatrix} + (ca+G) \begin{bmatrix} i_1 \\ i_1 + i_2 \end{bmatrix} \\
 & \text{can eliminate 2nd row} \quad 0 = (-g^2 + Gc - Gc) \begin{bmatrix} v_1 - v_2 \\ v_2 \end{bmatrix} + (ca+G)(i_1 + i_2)
 \end{aligned}$$