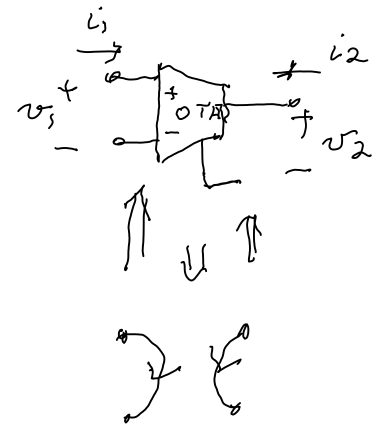
Note Title 9/2/2019

ENEE 610
Th 09/05/19

OTA
||
operational
transconductance
amplifier
= VCCS
||
voltage
controlled
current
source



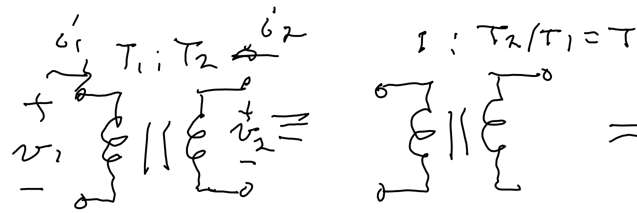
$i_1 = 0$

$i_2 = g_m v_1$

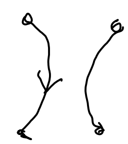
$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ g_m & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$$

$= Av = Bi$

transformer
(ideal)



$\Rightarrow$



$$\Rightarrow \begin{bmatrix} T & -1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 1 & -T \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$$

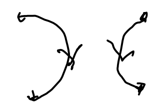
$Av = Bi$

$T v_1 = v_2$

$P_{in} = 0 \Rightarrow v_1 i_1 + v_2 i_2 = v_1 i_1 + T v_1 i_2 = 0 \Rightarrow i_1 + T i_2 = 0$

op-amp

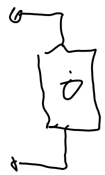
$\Rightarrow$



$$\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$$

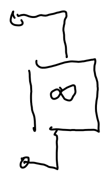
$i_1 = 0$

$v_1 = 0$



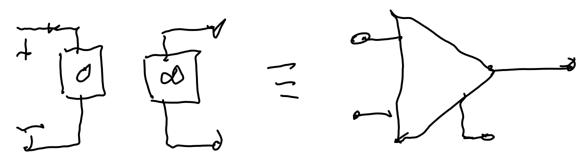
= nullator

$i = v = 0$



= norator

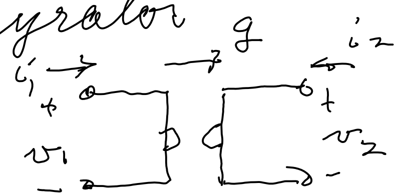
i, v arbitrary



i_2 & v_2 can be anything

i_1 & v_1 both 0 (a short & an open)

gyrator



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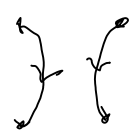


; $\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 0 & g \\ -g & 0 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = Yv = i$

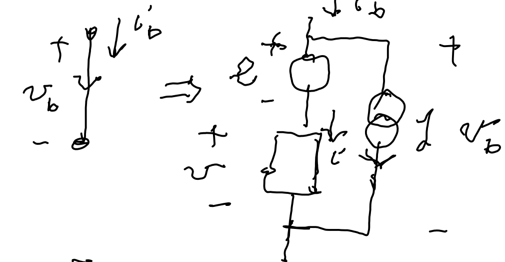
$Y = -Y^T$

power in $\quad v_1 i_1 + v_2 i_2 = v^T i = i^T v = \dfrac{v^T Y v + v^T Y^T v}{2} = \dfrac{v^T (Y + Y^T) v}{2} = 0$

$Av = Bi$ linear components

$v_b = \mathcal{C}^T v_t \qquad 0_t = \mathcal{C} i_b \qquad \Rightarrow \mathcal{C}\mathcal{J}^T = 0_{t \times l}$

$i_b = \mathcal{J}^T i_l \qquad 0_l = \mathcal{J} v_b$



$$v_b = e + v, \quad i_b = j + i \quad \Big\} \quad v = v_b - e, \quad i = i_b - j$$

$$A(\alpha)(v_b - e) = B(\alpha)(i_b - j)$$

$$\Downarrow$$

$$A(\alpha)\mathcal{C}^T v_t - A(\alpha) e = B(\alpha)\mathcal{J}^T i_l - B(\alpha) j$$

$$\underbrace{A(\alpha)\mathcal{C}^T v_t - B(\alpha)\mathcal{J}^T i_l}_{\text{circuit variables}} = \underbrace{A(\alpha) e - B(\alpha) j}_{\text{source terms}}$$

$$\left[A(\alpha)\mathcal{C}^T \;\vdots\; -B(\alpha)\mathcal{J}^T \right] \begin{bmatrix} v_t \\ \hline i_l \end{bmatrix} = A(\alpha) e - B(\alpha) j$$

$$x = \begin{bmatrix} v_t \\ i_l \end{bmatrix} \Rightarrow \text{semi-state vector}$$

(a b-vector)

If all components have admittance description

$$\left[\, Y_{b\times b}\, \mathcal{C}^T \;\vdots\; \underbrace{-\mathbb{1}_b\, \mathcal{J}^T}_{-\mathcal{J}^T} \,\right] x = A(x)e - B(x)j$$

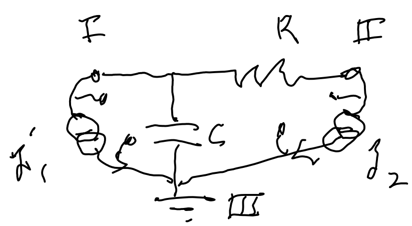
$$\Rightarrow \mathcal{C}\mathcal{J}^T = 0_{t\times \ell}$$

times $\mathcal{C}$

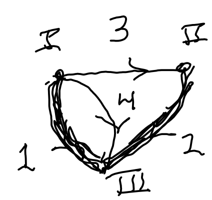
$$\mathcal{C}\, Y_{b\times b}\, \mathcal{C}^T v_t - \underset{0\times i_\ell}{0} = \mathcal{C}\,(Ae - Bj)$$

now have t equations

Ex:



$G = 1/R$



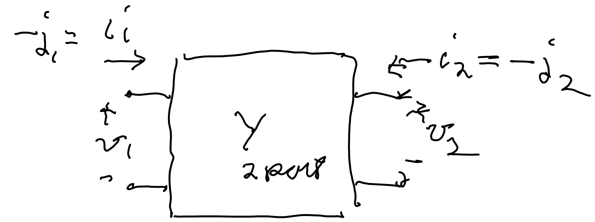
$$Av = Bi$$

$$\underset{Y_{b\times b}}{\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & G & 0 \\ 0 & 0 & 0 & sC \end{bmatrix}} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix} = \begin{bmatrix} & & \\ & \mathbb{1}_4 & \\ & & \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{bmatrix}$$

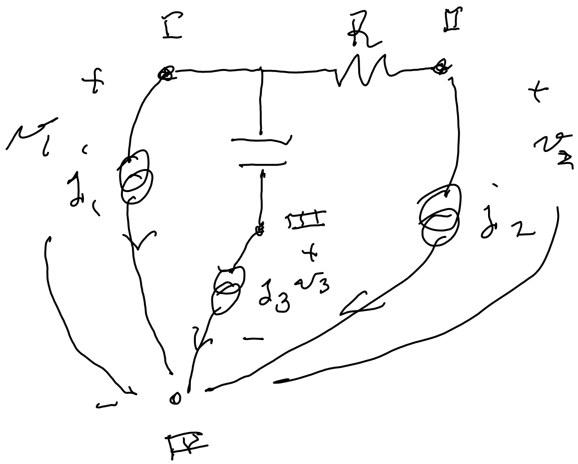
$$\mathcal{C} \Rightarrow \begin{array}{c} 1 \\ 2 \end{array}\begin{bmatrix} 0 \\ 0 \end{bmatrix} = \left[\begin{array}{cc|cc} 1 & 0 & 1 & 1 \\ 0 & 1 & -1 & 0 \end{array}\right] \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{bmatrix}$$

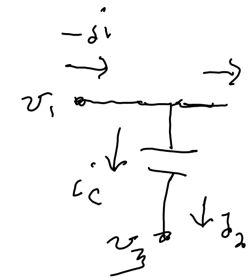
$$\mathcal{C}\, Y_{b\times b}\, \mathcal{C}^T = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & G & 0 \\ 0 & 0 & 0 & sC \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & -1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & G & sC \\ 0 & 0 & -G & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & -1 \\ 1 & 0 \end{bmatrix} = \underbrace{\begin{bmatrix} sC+G & -G \\ -G & G \end{bmatrix}}_{\text{nodal admittance}}$$

$$-\mathcal{C}\, j = -\begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} j_1 \\ j_2 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -j_1 \\ -j_2 \end{bmatrix}$$



$$Y = \begin{bmatrix} sC+G & -G \\ -G & G \end{bmatrix}$$





$$i_C = sC(v_1 - v_3)$$

$$i_R = G(v_1 - v_2)$$

$$-j_1 = i_C + i_R \Rightarrow j_1 = -sC v_1 + sC v_3 + G v_2 - G v_1$$

$$j_2 = +i_R = -G v_2 + G v_1$$

$$j_3 = i_C$$

$$Av = Bi$$

$$\begin{bmatrix} -(sC+G) & G & sC \\ +G & -G & 0 \\ sC & 0 & -sC \end{bmatrix}, \quad -\begin{bmatrix} \text{this} \end{bmatrix} = \underbrace{\begin{bmatrix} sC+G & -G & -sC \\ -G & G & 0 \\ -sC & 0 & sC \end{bmatrix}}_{\text{indefinite admittance}} = Y_{indef}$$

$$\begin{bmatrix} -j_1 \\ -j_2 \\ -j_3 \end{bmatrix} = \begin{bmatrix} sC+G & -G & -sC \\ -G & G & 0 \\ -sC & 0 & sC \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$$

ground v_3 node $\Rightarrow$ $v_3 = 0$

$$\begin{bmatrix} -j_1 \\ -j_2 \end{bmatrix} = \begin{bmatrix} sC+G & -G \\ -G & G \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \Rightarrow \begin{bmatrix} -j_1 \\ -j_2 \end{bmatrix} = \begin{bmatrix} sC+G & -G \\ -G & G \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$$

$$\begin{bmatrix} -j_3 \end{bmatrix} \quad \begin{bmatrix} -sC & G \end{bmatrix}$$