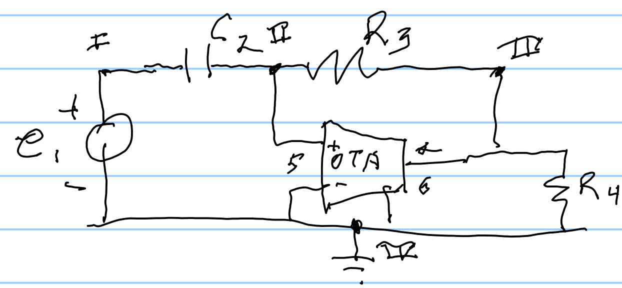
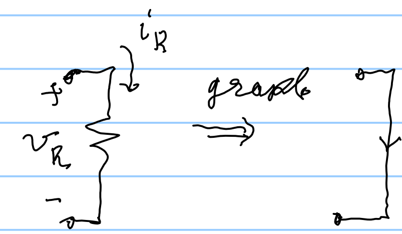
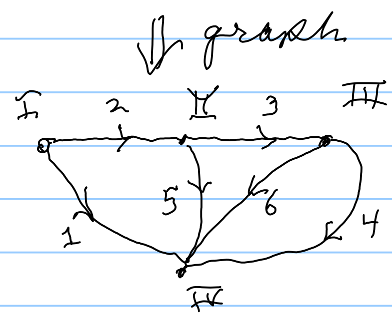
Note Title 8/29/2019

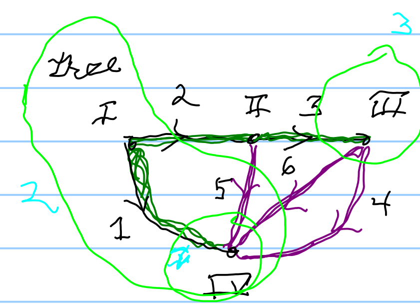
ENEE 610
Th 08/29/19







circuit graph



tree
branch
links
= cotree

KCL cut set equations

cut 1 : $0 = i_1 + 0 \cdot i_2 + 0 \cdot i_3 + i_4 + i_5 + i_6 \Rightarrow 0 = C\, i_b$

cut 2 : $0 = 0 \cdot i_1 + i_2 + 0 \cdot i_3 - i_4 - i_5 - i_6$

cut 3 : $0 = 0 \cdot i_1 + 0 \cdot i_2 + i_3 - i_4 + 0 \cdot i_5 - i_6$

$i_b = \begin{bmatrix} i_t \\ i_l \end{bmatrix} \begin{matrix} \} \text{tree} \\ \} \text{links} \end{matrix}$

$$t\left\{\underbrace{\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}}_{t\text{-vector}} = \left[\begin{array}{ccc|ccc} \overbrace{1 & 0 & 0}^{t} & \overbrace{1 & 1 & 1}^{t \times l} \\ 0 & 1 & 0 & -1 & -1 & -1 \\ \underbrace{0 & 0 & 1}_{1_t} & -1 & 0 & -1 \end{array}\right] \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ \hline i_4 \\ i_5 \\ i_6 \end{bmatrix} \begin{matrix} \left.\vphantom{\begin{matrix} i \\ i \\ i \end{matrix}}\right\} i_t \\ \left.\vphantom{\begin{matrix} i \\ i \\ i \end{matrix}}\right\} i_l \end{matrix}\right.$$

t = tree, $\#t = t = 3$

l = links, $\#l = l = 3$

n = nodes, $\#n = 4$

b = branch, $\#b = 6$

KVL tie set equations

links

$$l\left\{\begin{array}{c} 4 \\ 5 \\ 6 \end{array}\right. \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = \begin{array}{l} -v_1 + v_2 + v_3 \quad + v_4 + 0 \cdot v_5 + 0 \cdot v_6 \\ -v_1 + v_2 + 0 \cdot v_3 \quad 0 \cdot v_4 + v_5 + 0 \cdot v_6 \\ -v_1 + v_2 + v_3 \quad 0 \cdot v_4 \; 0 \cdot v_5 + v_6 \end{array} = \begin{bmatrix} \overbrace{-1 & 1 & 1}^{t} & 1 & 0 & 0 \\ -1 & 1 & 0 & 0 & 1 & 0 \\ -1 & 1 & 1 & \underbrace{0 & 0 & 1}_{1_l} \end{bmatrix} \underbrace{\begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ \hline v_4 \\ v_5 \\ v_6 \end{bmatrix}}_{v_b} \begin{matrix} \left.\vphantom{\begin{matrix} v \\ v \\ v \end{matrix}}\right\} v_t \\ \left.\vphantom{\begin{matrix} v \\ v \\ v \end{matrix}}\right\} v_l \end{matrix}$$

$0 = \mathcal{T} \cdot v_b$; $\quad \mathcal{T}$ = tie set matrix $= \left[\begin{array}{ccc|c} -1 & 1 & 1 & \\ -1 & 1 & 0 & 1_\ell \\ 1 & 1 & 1 & \end{array}\right] = \left[\begin{array}{c|c} -K^T & 1_\ell \end{array}\right]$

$0 = \mathcal{C} \cdot i_b$; $\quad \mathcal{C}$ = cutset Matrix $= \left[\begin{array}{c|ccc} & 1 & 1 & 1 \\ 1_t & -1 & -1 & -1 \\ & -1 & 0 & -1 \end{array}\right] = \left[\begin{array}{c|c} 1_t & K \end{array}\right]$

0 = power in from outside

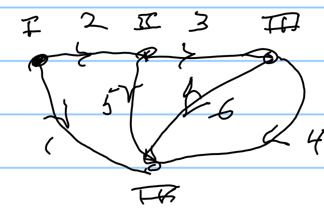
$= v_1 i_1 + v_2 i_2 \dots + v_6 i_6 = [v_1, v_2, \dots, v_6] \begin{bmatrix} i_1 \\ i_2 \\ \vdots \\ i_6 \end{bmatrix} = v_b^T \cdot i_b = 0$

$v_b = \mathcal{C}^T [v_t]$; $i_b = \mathcal{T}^T [i_\ell]$

$v_b^T \cdot i_b = (v_t^T \mathcal{C})(\mathcal{T}^T) i_\ell = \underset{\text{scalar}}{0}$ cancel i_ℓ & v_t as can independently choose every entry for the graph

$$C \cdot J^T = O_{t\times \ell} = \begin{bmatrix} 1_t & \vdots & K \end{bmatrix} \left(\begin{bmatrix} -K^T & \vdots & 1_\ell \end{bmatrix} \right)^T = \begin{bmatrix} 1_t & \vdots & K \end{bmatrix} \begin{bmatrix} -K & \vdots & 1_\ell \end{bmatrix} = \begin{bmatrix} -K + K \end{bmatrix} = 0$$

Incidence matrix



$$A = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 1 & 0 \\ 0 & 0 & -1 & 1 & 0 & 1 \\ -1 & 0 & 0 & -1 & -1 & -1 \end{bmatrix}, \ A \text{ is } n \times b$$

each column adds to zero

delete a row

$$A = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 1 & 0 \\ 0 & 0 & -1 & 1 & 0 & 1 \end{bmatrix},$$

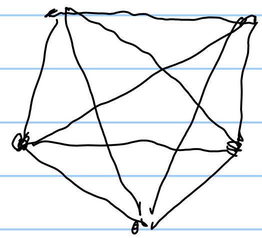
$$AA^T = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 1 & 0 \\ 0 & 0 & -1 & 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 3 & -1 \\ 0 & -1 & 3 \end{bmatrix}$$

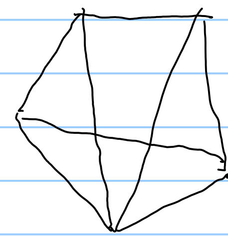
$$AA^T = (AA^T)^T$$

$$= D - A_g = L$$

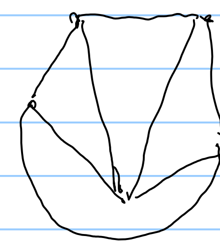
D = degree of nodes, L = Laplace matrix

$$\det(AA^T) = (18 + 0 + 0) - (2 + 3) = 18 - 5 = 13$$ = # of trees can form for the graph



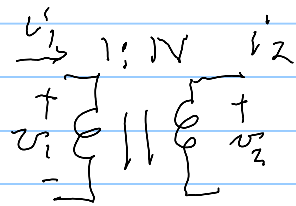


$\Rightarrow$



no crossing wires
=
planar graph

transformer

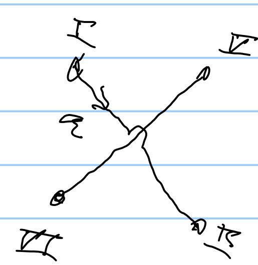


Power in $= v_1 i_1 + v_2 i_2$; $v_2 = N v_1$

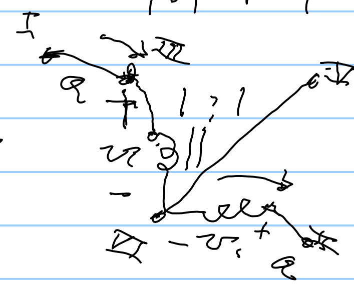
$\quad\;\; = v_1 i_1 + N v_1 i_2 \Rightarrow 0 = i_1 + N i_2 \Rightarrow i_1 = -N i_2$

0

if $N = 1$



$\Rightarrow$



has unchanged equations
allows nonplanar graphs
to become planar
(adds 2 of nodes & branches)