Solutions ETEE 610 final Fall 2018

#1.

$$y(s) = \frac{(s^2+1)(s^2+a)}{s(s^2+2)(s^2+b)} = \frac{k_0}{s} + \frac{k_1 s}{s^2+2} + \frac{k_2 s}{s^2+b} = \frac{\frac{a}{2b}}{s} + \frac{\frac{1}{2}\left(\frac{a-2}{b-2}\right)s}{s^2+2} + \frac{\left(\frac{b-1}{b}\right)\left(\frac{a-b}{2-b}\right)s}{s^2+b}$$

$$k_0 = y(s)\times s\Big|_{s=0} = \frac{(s^2+1)(s^2+a)}{(s^2+2)(s^2+b)}\Big|_{s=0} = \frac{(1)(a)}{(2)(b)} = \frac{a}{2b}$$

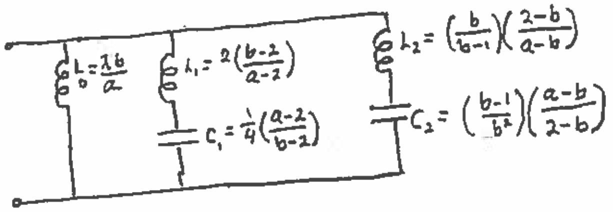
$$k_1 = y(s)\times\frac{(s^2+2)}{s}\Big|_{s^2=-2} = \frac{(s^2+1)(s^2+a)}{s^2(s^2+b)}\Big|_{s^2=-2} = \frac{(-1)(a-2)}{(-2)(b-2)} = \frac{1}{2}\left(\frac{a-2}{b-2}\right)$$

$$k_2 = y(s)\times\frac{(s^2+b)}{s}\Big|_{s^2=-b} = \frac{(s^2+1)(s^2+a)}{s^2(s^2+2)}\Big|_{s^2=-b} = \frac{(1-b)}{(-b)}\left(\frac{a-b}{2-b}\right) = \left(\frac{b-1}{b}\right)\left(\frac{a-b}{2-b}\right)$$

This requires poles and zeroes to alternate thus & on $j\omega$ axis $(a, b \geq 0)$

$\therefore$ $0 \leq a < b < 1$, $2 \leq a < b$ or $0 \leq a = b$ real

The generic circuit is



which reduces when $a = 2$ or $a = b$ or ($b = 1$ with $0 \leq a \leq 2$)

#2. $P(s) = \text{Ev}\,P + \text{Od}\,P = (2s^6+9s^4+12s^2+4) + (s^7+6s^5+12s^3+8s)$

choose: $y(s) = \dfrac{\text{Od}\,P}{\text{Ev}\,P}$ ⇒ continued fraction about ∞

$$\begin{array}{r|l} & \tfrac{1}{2}s \\ 2s^6+9s^4+12s^2+4 & s^7+6s^5+12s^3+8s \\ & s^7+\tfrac{9}{2}s^5+6s^3\quad 2s \\ \hline & \tfrac{12-9}{2}s^5=\tfrac{3}{2}s^5+6s^3+6s \end{array}$$

$$\begin{array}{r|l} & \tfrac{4}{3}s \\ \tfrac{3}{2}s^5+6s^3+6s & 2s^6+9s^4+12s^2+4 \\ & 2s^6+8s^4+8s^2 \\ \hline & s^4+4s^2+4 \end{array}$$

$$\begin{array}{r|l} & \tfrac{3}{2}s \\ s^4+4s^2+4 & \tfrac{3}{2}s^5+6s^3+6s \\ & \tfrac{3}{2}s^5+6s^3+6s \\ \hline & 0\quad 0\quad 0 \end{array}$$

∴ there is a common factor in Ev P & Od P

to find

$$\frac{\text{Od}\,P}{\text{Ev}\,P} = \frac{1}{2}s + \cfrac{1}{\frac{4}{3}s + \cfrac{1}{\frac{3}{2}s}} = \frac{1}{2}s + \frac{\frac{3}{2}s}{2s^2+1} = \frac{s^3+\frac{1}{2}s+\frac{3}{2}s}{2s^2+1}$$

$$= \frac{s^3+2s}{2s^2+1} \Rightarrow P_1 = \text{Ev}\,P_1 + \text{Od}\,P_1 = (2s^2+1)+(s^3+2s) = s^3+2s^2+2s+1$$

and $P = P_1 \cdot Q$, Q = common factor ⇒ $Q = \dfrac{P}{P_1}$, divide P by P_1

$$\begin{array}{r|l} & s^4+4s^2+4 \\ s^3+2s^3+2s+1 & s^7+2s^6+6s^5+9s^4+12s^3+12s^2+8s+4 \\ & s^7+2s^6+2s^5+s^4 \\ \hline & 4s^5+8s^4+12s^3+12s^2+8s+4 \\ & 4s^5+8s^4+8s^3+4s^2 \\ \hline & 4s^3+8s^2+8s+4 \\ & 4s^3+8s^2+8s+4 \\ \hline \end{array}$$

$$\Rightarrow Q = s^4+4s^2+4 = (s^2+2)^2$$

$$\therefore\ P(s) = \underset{\text{(only in LHP)}}{(\text{Hurwitz})} \times \underbrace{\left(\begin{array}{c}\text{double zero}\\ \text{on } j\omega \text{ axis}\end{array}\right)}_{(s^2+2)^2} \Rightarrow \text{unstable} \Rightarrow P(s) \text{ is not Hurwitz}$$

(or strictly Hurwitz)

#3. a) $\begin{bmatrix}-m_v & 1\\ 0 & 0\end{bmatrix}\begin{bmatrix}v_1\\ v_2\end{bmatrix}=\begin{bmatrix}0 & 0\\ -m_i & 1\end{bmatrix}\begin{bmatrix}i_1\\ i_2\end{bmatrix}\equiv Av=Bi$ as $\det A=\det B=0$ no $Y=B^{-1}A$ or $Z=A^{-1}B$ exists, all real m_v, m_i

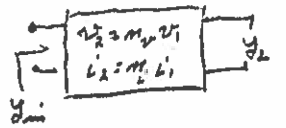
For $S=(B+A)^{-1}(B-A)=\begin{bmatrix}-m_v & 1\\ -m_i & 1\end{bmatrix}^{-1}\begin{bmatrix}m_v & -1\\ -m_i & 1\end{bmatrix}=\frac{1}{m_i-m_v}\begin{bmatrix}1 & -1\\ m_i & -m_v\end{bmatrix}\begin{bmatrix}m_v & -1\\ -m_i & 1\end{bmatrix}=\frac{1}{m_i-m_v}\begin{bmatrix}-(m_v+m_i) & -2\\ 2m_vm_i & -(m_v+m_i)\end{bmatrix}$

which exists for all m_v & m_i except when $m_i=m_v$

b) This is passive if $\mathcal{E}(t)=\int_{-\infty}^{t}p(\tau)d\tau\geq 0$ where $p(t)=v^T(t)i(t)$

here $p(t)=v_1i_1+v_2i_2=v_1i_1+m_vm_iv_1i_1=(1+m_vm_i)v_1i_1$

as v_1 & i_1 are arbitrary they can have $\text{sign}(v_1i_1)=-\text{sign}(1+m_vm_i)$ except when $1+m_vm_i\equiv 0$ in which case $\mathcal{E}(t)$ can be <0 unless $m_vm_i=-1$ when $\mathcal{E}(t)=0$. Therefore the 2-port is passive and lossless only when $m_i=-1/m_v$ [which is an ideal transformer]

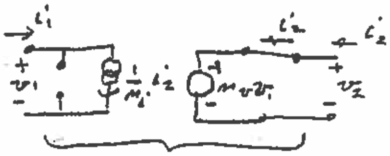
c) 

$y_{in}=\frac{i_1}{v_1}=\frac{(1/m_i)i_2}{(1/m_v)v_2}=\frac{m_v}{m_i}\left(\frac{-i_L}{v_L}\right)=\underline{-\frac{m_v}{m_i}y_L=y_{in}}$

If m_v & m_i have the same sign y_{in} is the scaled negative of y_L & the 2-port is a negative impedance converter. If the 2-port is passive it is a transformer & $y_{in}=(m_v)^2y_L$

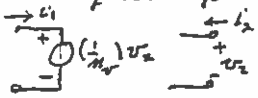
d) $v_2=m_vv_1$ is a VCVS & $i_2=m_ii_1$ is a CCCS, better as $i_1=\frac{1}{m_i}i_2$

Using the latter form of $i_1=\frac{1}{m_i}i_2$ allows the connection if $m_i\neq 0$



2port $v_2=m_vv_1$, $i_1=\left(\frac{1}{m_i}\right)i_2$ (which holds if $m_i\neq 0, m_v=0$)

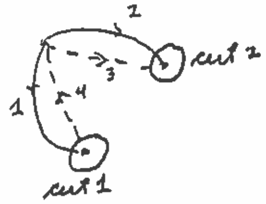
If $m_i=0$ then $i_2=0$ and i_1 can be in a short requiring use of $v_1=(1/m_v)v_2$



If m_i & $m_v=0$ we have $v_2=i_2=0$ & v_1 & i_1 arbitrary or a nullator @ port 2 & a norator @ port 1

case $m_v=m_i=0$

#4 a)



1 & 2 as tree

Cut set $\begin{array}{c} \text{cut 1} \\ \text{cut 2} \end{array} \underset{\text{KCL}}{\begin{bmatrix} 0 \\ 0 \end{bmatrix}} = \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{bmatrix} \equiv \underline{0}_t = \mathcal{C}\, i_b$

$$\Rightarrow \mathcal{C}_1 = \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 1 & 0 \end{bmatrix}$$

Tie set $\begin{array}{c} \text{link 3} \\ \text{link 4} \end{array} \underset{\text{KVL}}{\begin{bmatrix} 0 \\ 0 \end{bmatrix}} = \begin{bmatrix} 0 & -1 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix} \equiv \underline{0}_\ell = \mathcal{T} v_b$

$$\Rightarrow \mathcal{T}_1 = \begin{bmatrix} 0 & -1 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix}$$

b) 3 & 4 as tree

$$\begin{array}{c} \text{cut 1} \\ \text{cut 2} \end{array} \underset{\text{KCL}}{\begin{bmatrix} 0 \\ 0 \end{bmatrix}} = \begin{bmatrix} 0 & 1 & 1 & 0 \\ -1 & 0 & 0 & 1 \end{bmatrix} i_b \Rightarrow \mathcal{C}_2 = \begin{bmatrix} 0 & 1 & 1 & 0 \\ -1 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{array}{c} \text{link 1} \\ \text{link 2} \end{array} \underset{\text{KVL}}{\begin{bmatrix} 0 \\ 0 \end{bmatrix}} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & -1 & 0 \end{bmatrix} v_b \Rightarrow \mathcal{T}_2 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & -1 & 0 \end{bmatrix}$$

c) $\mathcal{C}_2 = R_c \mathcal{C}_1 \Rightarrow \begin{bmatrix} 0 & 1 & 1 & 0 \\ -1 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} r_{11} & r_{12} \\ r_{21} & r_{22} \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 1 & 0 \end{bmatrix}$

R multiplies row 1 by $\underset{\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}}{\underset{\shortparallel}{-1}}$ & then $\underset{\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}}{\underset{\shortparallel}{\text{permutes rows}}}$

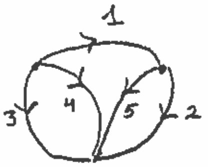
$$\therefore\ R_c = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$$
$$= \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

For $\mathcal{T}_2 = R_t \mathcal{T}_1 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & -1 & 0 \end{bmatrix} = \begin{bmatrix} \ \Downarrow\ \end{bmatrix} \begin{bmatrix} 0 & -1 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix}$

row 1 × (-1)
then permute rows

$$\Rightarrow R_t = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$$
$$= \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \equiv R_c$$

#5. Graph



tree: 1 & 2
links: 3, 4, 5

Cutset $\begin{bmatrix}0\\0\end{bmatrix}=\left[\begin{array}{cc|ccc}1&0&1&1&0\\0&1&1&1&1\end{array}\right]i_b$; $v_b=\mathcal{C}^T v_t$

Tie set $\begin{bmatrix}0\\0\\0\end{bmatrix}=\left[\begin{array}{cc|ccc}-1&-1&1&0&0\\-1&-1&0&1&0\\0&-1&0&0&1\end{array}\right]v_b$; $i_b=\mathcal{T}^T i_\ell$

$Av=Bi$

$$\begin{bmatrix}sC_1&0&0&0&0\\0&sC_2&0&0&0\\0&0&1&0&0\\0&0&0&0&g\\0&0&0&-g&0\end{bmatrix}v=\begin{bmatrix}1&0&0&0&0\\0&1&0&0&0\\0&0&0&0&0\\0&0&0&1&0\\0&0&0&0&1\end{bmatrix}i \qquad e=\begin{bmatrix}0\\0\\e_3\\0\\0\end{bmatrix},\ j=\begin{bmatrix}0\\0\\0\\0\\0\end{bmatrix}$$

$v_b=v+e$, $i_b=i+j=i$

$\Downarrow \qquad\qquad \Downarrow$

$v=v_b-e \qquad i=\mathcal{T}^T i_\ell \implies A\mathcal{C}^T v_t-Ae=B\mathcal{T}^T i_\ell \implies A\mathcal{C}^T v_t-B\mathcal{T}^T i_\ell=Ae$

$=\mathcal{C}^T v_t-e$

$$A\mathcal{C}^T=\begin{bmatrix}sC_1&&&&\\&sC_2&&&\\&&1&&\\&&&0&g\\&&&-g&0\end{bmatrix}\begin{bmatrix}1&0\\0&1\\1&1\\1&1\\0&1\end{bmatrix}=\begin{bmatrix}sC_1&0\\0&sC_2\\1&1\\0&g\\-g&-g\end{bmatrix},\ B\mathcal{T}^T=\begin{bmatrix}1&&&&\\&1&&&\\&&0&&\\&&&1&\\&&&&1\end{bmatrix}\begin{bmatrix}-1&-1&0\\-1&-1&-1\\1&0&0\\0&1&0\\0&0&1\end{bmatrix}=\begin{bmatrix}-1&-1&0\\-1&-1&-1\\0&0&0\\0&1&0\\0&0&1\end{bmatrix}$$

$$Ae=\begin{bmatrix}sC_1&&&&\\&sC_2&&&\\&&1&&\\&&&0&g\\&&&-g&0\end{bmatrix}\begin{bmatrix}0\\0\\e_3\\0\\0\end{bmatrix}=\begin{bmatrix}0\\0\\e_3\\0\\0\end{bmatrix};\ x=\begin{bmatrix}v_1\\v_2\\i_3\\i_4\\i_5\end{bmatrix} \implies \left[A\mathcal{C}^T \,\vdots\, -B\mathcal{T}^T\right]\begin{bmatrix}v_t\\i_\ell\end{bmatrix}=Ae$$

$$\therefore \underbrace{\begin{bmatrix}sC_1&0&0&0&0\\0&sC_2&0&0&0\\0&0&0&0&0\\0&0&0&0&0\\0&0&0&0&0\end{bmatrix}}_{sE}x=\underbrace{\begin{bmatrix}0&0&-1&-1&0\\0&0&-1&-1&-1\\-1&-1&0&0&0\\0&-g&0&1&0\\g&g&0&0&1\end{bmatrix}}_{A}x+\underbrace{\begin{bmatrix}0\\0\\1\\0\\0\end{bmatrix}}_{B}e_3 \implies sEx=Ax+Be_3$$

as $y=x_2 \implies v_{c_2}=\underbrace{[0\ 1\ 0\ 0\ 0]}_{C}x$

b) $Y_{2\text{-port}}=\begin{bmatrix}sC_1&-sC_1+g\\-sC_1-g&sC_1\end{bmatrix} \implies y_{in}=y_{11}-y_{12}\cdot\dfrac{1}{y_{22}+y_L}\cdot y_{21}=\dfrac{y_{11}y_L+\det y}{y_{22}+y_L}$

$=(sC_1\cdot sC_2+g^2)/[sC_1+sC_2]$

$v_{c_2}/e_3=-y_{21}/(y_L+y_{22}=\dfrac{\overset{+}{-}(\overset{+}{-}sC_1\overset{+}{-}g)}{sC_1+sC_2}$ from $i_2=-y_Lv_2=y_{21}v_1+y_{22}v_2$

c) For the semistate eqs. need to calculate $v_2=v_{c_2}=C(sE-A)^{-1}Be_3$ for just the voltage gain, along with finding $sEx=Ax+Bu$ & $y=Cx$

so using 2-port Y directly is much simpler though less general