ENEE 610

Electrical Network Theory

HomeWork # 07

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Problem 01

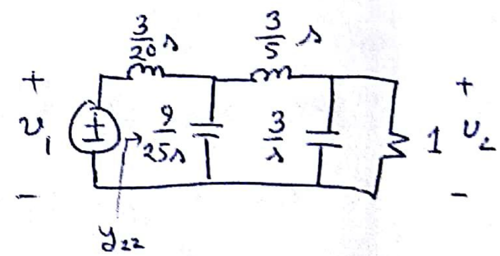
a) Here, $\dfrac{V_2}{V_1} = \dfrac{K}{s^4+3s^3+8s^2+9s+12} = \dfrac{\dfrac{K}{3s^3+9s}}{1+\dfrac{s^4+8s^2+12}{3s^3+9s}}$

So, $y_{22} = \dfrac{s^4+8s^2+12}{3s^3+9s}$

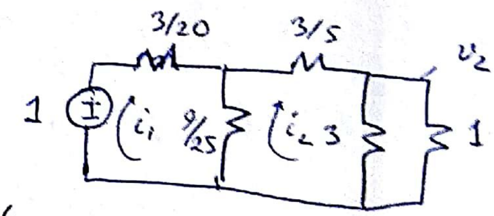
Now,

$$3s^3+9s \,)\, s^4+8s^2+12 \,(\, \tfrac{1}{3}s$$
$$s^4+3s^2$$
$$5s^2+12 \,)\, 3s^3+9s \,(\, \tfrac{3}{5}s$$
$$3s^3+\tfrac{36}{5}s$$
$$\tfrac{9}{5}s \,)\, 5s^2+12 \,(\, \tfrac{25}{9}s$$
$$5s^2$$
$$12 \,)\, \tfrac{9}{5}s \,(\, \tfrac{3}{20}s$$
$$\tfrac{9}{5}s$$
$$0$$

So, $\boxed{y_{22}(s) = \dfrac{1}{3}s + \dfrac{1}{\dfrac{3}{5}s + \dfrac{1}{\dfrac{25}{9}s + \dfrac{1}{\dfrac{3}{20}s}}}}$



Now, to evaluate K, let $s=1$ & $v_1 = 1$.



Solving using KVL, $i_1 = \dfrac{76}{33}$, $i_2 = \dfrac{16}{33}$

So, $V_2 = \dfrac{3}{4} i_2 = \dfrac{4}{11}$

again, $\dfrac{V_2}{V_1} = \dfrac{K}{1+3+8+9+12} \Rightarrow V_2 = \dfrac{K}{33}$

Equating, $\boxed{K = 12}$

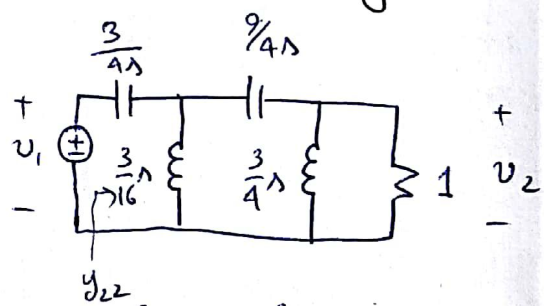
(b) Here, $\dfrac{V_2}{V_1} = \dfrac{Ks^4}{s^4+3s^3+8s^2+9s+12} = \dfrac{\dfrac{Ks^4}{3s^3+9s}}{1+\dfrac{s^4+8s^2+12}{3s^3+9s}}$

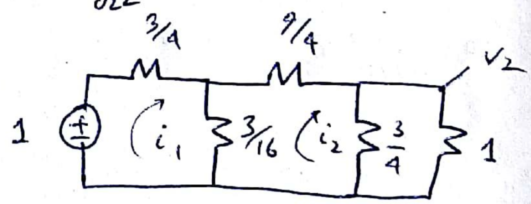
So, $y_{22} = \dfrac{s^4+8s^2+12}{3s^3+9s}$

Now,

$$9s+3s^3\,\big)\,12+8s^2+s^4\,\big(\,\frac{4}{3s}$$
$$\underline{12+4s^2}$$
$$4s^2+s^4\,\big)\,9s+3s^3\,\big(\,\frac{9}{4s}$$
$$\underline{9s+\frac{9}{4}s^3}$$
$$\frac{3}{4}s^3\,\big)\,4s^2+s^4\,\big(\,\frac{16}{3s}$$
$$\underline{4s^2}$$
$$s^4\,\big)\,\frac{3}{4}s^3\,\big(\,\frac{3}{4s}$$
$$\underline{\frac{3}{4}s^3}$$
$$0$$

So, $\boxed{y_{22}(s) = \dfrac{4}{3s} + \dfrac{1}{\dfrac{9}{4s}+\dfrac{1}{\dfrac{16}{3s}+\dfrac{1}{\dfrac{3}{4s}}}}}$





Now, let, $v_1 = 1$, $s = 1$

Solving using KVL,

$i_1 = \dfrac{107}{99}$, $i_2 = \dfrac{7}{99}$

So, $V_2 = \dfrac{3}{7} \times \dfrac{7}{99} = \dfrac{1}{33}$

Again, $V_2 = \dfrac{K}{33} \Rightarrow \boxed{K = 1}$

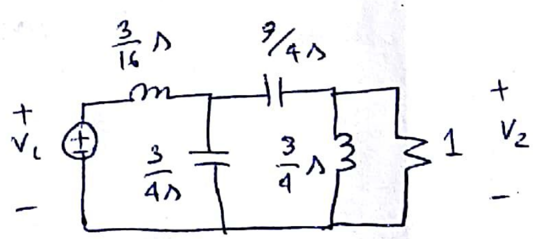
(c) Here, $\frac{V_2}{V_1} = \frac{Ks^2}{s^4+3s^3+8s^2+9s+12} = \frac{\frac{Ks^2}{3s^3+9s}}{1+\frac{s^4+8s^2+12}{3s^3+9s}}$

So, $y_{22} = \dfrac{s^4+8s^2+12}{3s^3+9s}$

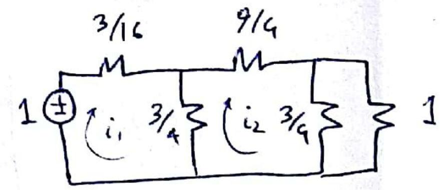
Now,

$$9s+3s^3 \,)\, 12+8s^2+s^4 \,(\, \frac{4}{3s}$$
$$12+4s^2$$
$$4s^2+s^4 \,)\, 9s+3s^3 \,(\, \frac{9}{4s}$$
$$9s+\frac{9}{4}s^3$$
$$\frac{3}{4}s^3 \,)\, s^4+4s^2 \,(\, \frac{4}{3}s$$
$$s^4$$
$$4s^2 \,)\, \frac{3}{4}s^3 \,(\, \frac{3}{16}s$$
$$\frac{3}{4}s^3$$
$$0$$

So, $\boxed{y_{22}(s) = \frac{4}{3s} + \cfrac{1}{\frac{9}{4s} + \cfrac{1}{\frac{4}{3}s + \cfrac{1}{\frac{3}{16}s}}}}$



Now, let, $V_1 = 1$, $s = 1$



Solving, we get,

$i_1 = \frac{128}{99}$, $i_2 = \frac{28}{99}$

So, $V_2 = \frac{3}{7} \times \frac{28}{99} = \frac{4}{33}$

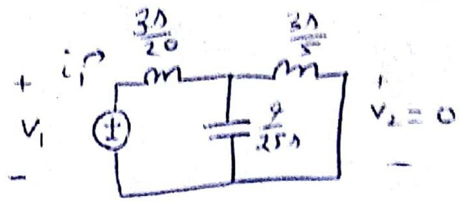
But, $V_2 = \frac{K}{33} \Rightarrow \boxed{K = 4}$

(d) For part (a), $\frac{V_2}{V_1} = \frac{12}{s^4+3s^3+8s^2+9s+12} = \frac{\frac{12}{3s^3+9s}}{1+\frac{s^4+9s^2+12}{3s^3+9s}} = \frac{y_{21}}{1+y_{22}}$

So, we get, $y_{21} = -\frac{12}{3s^3+9s}$, $y_{22} = \frac{s^4+9s^2+12}{3s^3+9s}$

From the symmetry condition, $y_{12} = y_{21} = -\frac{12}{3s^3+9s}$

Now, $y_{11} = \left.\frac{i_i}{v_i}\right|_{v_2=0}$



Here, $z_{11} = \frac{3s}{20} + \left(\frac{9}{25s} \parallel \frac{3s}{5}\right)$

$= \frac{3s}{20} + \frac{\frac{9}{25s} \times \frac{3s}{5}}{\frac{9}{25s} + \frac{3s}{5}}$

$= \frac{3s}{20} + \frac{27s}{5(15s^2+9)}$

$= \frac{3s^3+9s}{20s^2+12}$

So, $y_{11} = \frac{20s^2+12}{3s^3+9s}$

So, $\boxed{Y(s) = \begin{bmatrix} \frac{20s^2+12}{3s^3+9s} & -\frac{12}{3s^3+9s} \\ -\frac{12}{3s^3+9s} & \frac{s^4+9s^2+12}{3s^3+9s} \end{bmatrix}}$

Here, $Y(s) + Y^T(-s) = 0$

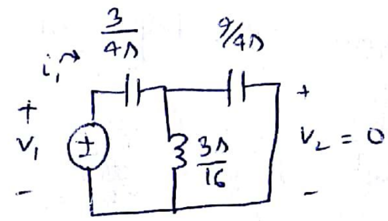
So, $Y(s)$ is a lossless positive real system.

For part (b), $\frac{V_2}{V_1} = \frac{s^4}{s^4+3s^3+8s^2+9s+12} = \frac{\frac{s^4}{3s^3+9s}}{1+\frac{s^4+8s^2+12}{3s^3+9s}} = \frac{-y_{21}}{1+y_{22}}$

So, $y_{21} = -\frac{s^4}{3s^3+9s}$, $y_{22} = \frac{s^4+8s^2+12}{3s^3+9s}$

From symmetry condition, $y_{12} = y_{21} = -\frac{s^4}{3s^3+9s}$

Now, $y_{11} = \left.\frac{i_1}{v_1}\right|_{V_2=0}$



Now, $z_{11} = \frac{3}{4s} + \left(\frac{3s}{16} \parallel \frac{9}{4s}\right)$

$= \frac{3}{4s} + \frac{\frac{3}{16}\times\frac{9}{4}}{\frac{3s}{16}+\frac{9}{4s}} = \frac{3}{4s} + \frac{27s}{12s^2+144}$

$= \frac{3s^2+9}{s^3+12s}$

So, $y_{11} = \frac{s^4+12s^2}{3s^3+9s}$

So, $Y(s) = \boxed{\begin{bmatrix} \frac{s^4+12s^2}{3s^3+9s} & -\frac{s^4}{3s^3+9s} \\ -\frac{s^4}{3s^3+9s} & \frac{s^4+8s^2+12}{3s^3+9s} \end{bmatrix}}$

Here, $Y(s) + Y^T(-s) = 0$, so it is a lossless positive real system.

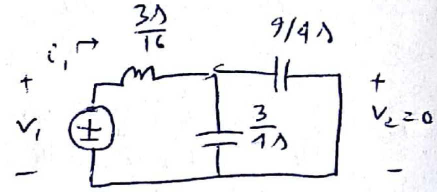
Finally, for part ©,

$$\frac{V_2}{V_1} = \frac{4s^2}{s^4+3s^3+8s^2+9s+12} = \frac{\dfrac{4s^2}{3s^3+9s}}{1+\dfrac{s^4+8s^2+12}{3s^3+9s}} = \frac{-y_{21}}{1+y_{22}}$$

So, $y_{21} = -\dfrac{4s^2}{3s^3+9s}$, $y_{22} = \dfrac{s^4+8s^2+12}{3s^3+9s}$

From symmetry condition, $y_{12} = y_{21} = -\dfrac{4s^2}{3s^3+9s}$

Now, $y_{11} = \left.\dfrac{i_1}{v_1}\right|_{V_2=0}$



where, $z_{11} = \dfrac{3s}{16} + \left(\dfrac{3}{4s} \,||\, \dfrac{9}{4s}\right)$

$$= \frac{3s}{16} + \frac{\dfrac{3\times 9}{16s^2}}{\dfrac{3}{4s}+\dfrac{9}{4s}}$$

$$= \frac{3s}{16} + \frac{9}{16s}$$

$$= \frac{3s^2+9}{16s}$$

So, $y_{11} = \dfrac{16s^2}{3s^3+9s}$

So, $$Y(s) = \begin{bmatrix} \dfrac{16s^2}{3s^3+9s} & -\dfrac{4s^2}{3s^3+9s} \\ -\dfrac{4s^2}{3s^3+9s} & \dfrac{s^4+8s^2+12}{3s^3+9s} \end{bmatrix}$$

Here, $Y(s) + Y^T(-s) = 0$, So, the system is a lossless positive real one.

Problem 02

(a) Here, $Z(s) = \dfrac{3s(s^2+4)}{s^2+2} \Rightarrow Y(s) = \dfrac{s^2+2}{3s(s^2+4)} = \dfrac{i(s)}{v(s)}$

~~Here~~, let, $x_1 = \dfrac{1}{3s(s^2+4)} v(s)$

$\Rightarrow v(s) = 3s^3 x_1(s) + 12 s x_1(s)$

Now, let, $x_2 = s x_1 = \dot{x}_1$

& $x_3 = s x_2 = s^2 x_1 = \ddot{x}_1$

then, $v = 3\dot{x}_3 + 12 x_2 \Rightarrow \dot{x}_3 = \frac{1}{3} v - 4x_2$

So, we can write,

$$\begin{aligned}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= x_3 \\
\dot{x}_3 &= -4x_2 + \tfrac{1}{3} v \\
i &= 2x_1 + x_3
\end{aligned}$$

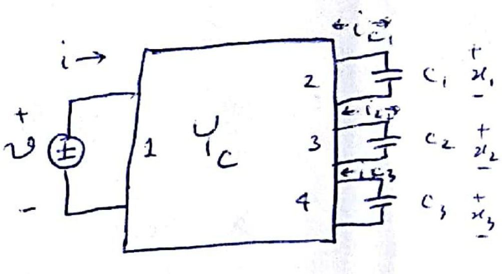
So, we can write state-space realization as :-

$\dot{x} = Ax + Bv \Rightarrow \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -4 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1/3 \end{bmatrix} v$

& $i = Cx + Dv \Rightarrow i = \begin{bmatrix} 2 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + [0] v$

Now, let, $-C_{int}\dot{x} = -\begin{bmatrix} C_1 & 0 & 0 \\ 0 & C_2 & 0 \\ 0 & 0 & C_3 \end{bmatrix}\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = -\begin{bmatrix} C_1\dot{x}_1 \\ C_2\dot{x}_2 \\ C_3\dot{x}_3 \end{bmatrix}$ is the current, through capacitors of value $C_1, C_2 \,\&\, C_3$, i_{int}.

Then, we can synthesize the network as:-



Here, x is the potential drop across the capacitors. Then, the system can be written as,

$$\begin{bmatrix} i \\ i_{C_1} \\ i_{C_2} \\ i_{C_3} \end{bmatrix} = Y_c \begin{bmatrix} v \\ x_1 \\ x_2 \\ x_3 \end{bmatrix} = \left[\begin{array}{c|c} D & C \\ \hline -C_{int}B & -C_{int}A \end{array}\right]\begin{bmatrix} v \\ x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Here, Y_c is the coupling admittance network. Choosing,

$C_{int} = 1_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, $Y_c = \begin{bmatrix} D & C \\ -B & -A \end{bmatrix}$

So,

$$\begin{bmatrix} i \\ i_{C_1} \\ i_{C_2} \\ i_{C_3} \end{bmatrix} = \underbrace{\left[\begin{array}{c|ccc} 0 & 2 & 0 & 1 \\ \hline 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \\ -1/3 & 0 & 4 & 0 \end{array}\right]}_{Y_c}\begin{bmatrix} v \\ x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

So, we can synthesize the coupling network using parallel combinations of OTA's, & set all capacitors as unity to get the output current at the input terminal.

Here, $y(s)$ has a pole at infinity, which means the system is an LPR & can be synthesized.

(b) Here, $\frac{V_2}{V_1} = \frac{s^2 - as + b}{s^2 + as + b}$

Let, $x_1 = \frac{1}{s^2 + as + b} V_1$

$\Rightarrow V_1 = (s^2 + as + b) x_1 = \ddot{x}_1 + a\dot{x}_1 + bx_1$

again, let, $x_2 = \dot{x}_1 \Rightarrow V_1 = \dot{x}_2 + ax_2 + bx_1$

also, $V_2 = (s^2 - as + b) x_1 = \ddot{x}_1 - a\dot{x}_1 + bx_1$

$= \dot{x}_2 - ax_2 + bx_1 = V_1 - 2ax_2$

So, we can write,

$V_2 = -2ax_2 + V_1$

$\dot{x}_1 = x_2$

$\dot{x}_2 = -bx_1 - ax_2 + V_1$

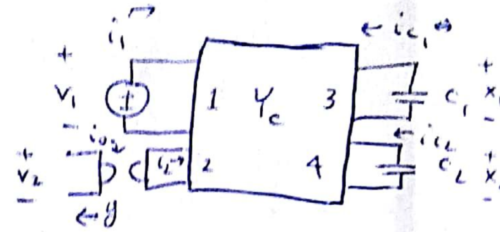
So, state space realization,

$\dot{x} = Ax + BV_1 \Rightarrow \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -b & -a \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} V_1$

& $V_2 = Cx + DV_1 \Rightarrow V_2 = \begin{bmatrix} 0 & -2a \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + [1] V_1$

Now, to synthesize the circuit, we can use a gyrator of gain g to get the output, & we can use intermediate capacitor terminals such that $-C_{int}\dot{x} = i_{int}$; where x is the voltage across capacitors C_1 & C_2.

It can be visualized as the figure that follows:



Then,

$$\begin{bmatrix} i_1 \\ i_2 \\ i_{c_1} \\ i_{c_2} \end{bmatrix} = Y_c \begin{bmatrix} v_1 \\ v_2 \\ x_1 \\ x_2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} i_1 \\ i_2 \\ i_{int} \end{bmatrix} = \begin{bmatrix} G_{11} & -\frac{1}{g} D^T & B^T C_{int}^T \\ \frac{1}{g} D & G_{22} & \frac{1}{g} C \\ -C_{int} B & -\frac{1}{g} C^T & -C_{int} A \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ \cancel{x_{int}} \; x \end{bmatrix}$$

For simplicity, choosing $g = 1$, $C_{int} = 1_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, & $G_{11} = G_{22} = 0$, we get, the coupling admittance,

$$Y_c = \begin{bmatrix} 0 & -D & B^T \\ D & 0 & C \\ -B & -C^T & -A \end{bmatrix}$$

So,

$$\begin{bmatrix} i_1 \\ i_2 \\ i_{c_1} \\ i_{c_2} \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & -1 & 0 & 1 \\ 1 & 0 & 0 & -2a \\ 0 & 0 & 0 & -1 \\ -1 & 2a & b & a \end{bmatrix}}_{Y_c} \begin{bmatrix} v_1 \\ v_2 \\ x_1 \\ x_2 \end{bmatrix}$$

So, we can synthesize this all-pass circuit using parallel combinations of OTA's & gyrators, & set the capacitor & output gyrator as unity to design the required network.

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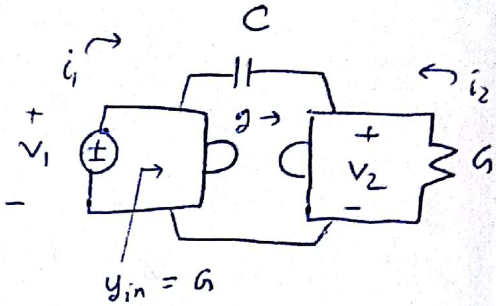
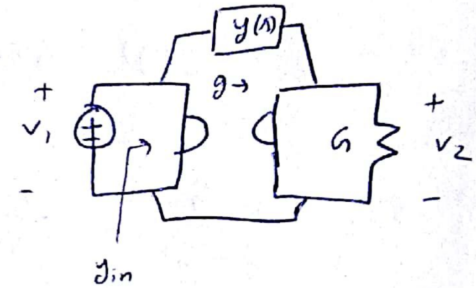
Problem 03

a) we know, in Richards' section,



$$Y_R = \begin{bmatrix} sC & -sC+g \\ -sC-g & sC \end{bmatrix}$$

Now, replacing sC with $y(s)$, we get,



$$Y_R = \begin{bmatrix} y & -y+g \\ -y-g & y \end{bmatrix}$$

So, $\Delta Y_R = y^2 - (y^2 - g^2) = g^2$

Now, $y_{in} = y_{11} - \dfrac{y_{12}\,y_{21}}{y_{22}+y_L} = \dfrac{\Delta Y_R + y_{11}\,y_L}{y_{22}+y_L}$

$$= \frac{g^2 + yG}{y+G}$$

But, for Richards' section, $G = |g| \Rightarrow g^2 = G^2$

So, $y_{in} = \dfrac{G^2 + yG}{y+G} = G = y_L$

So, this circuit is still an all-pass constant-R network.

Now, we need to determine the $y(s)$ required such that,

$$\frac{V_2}{V_1} = \frac{s^2 - as + b}{s^2 + as + b}$$

We know, for Richards' section, $\dfrac{V_2}{V_1} = \dfrac{sC - G}{sC + G}$

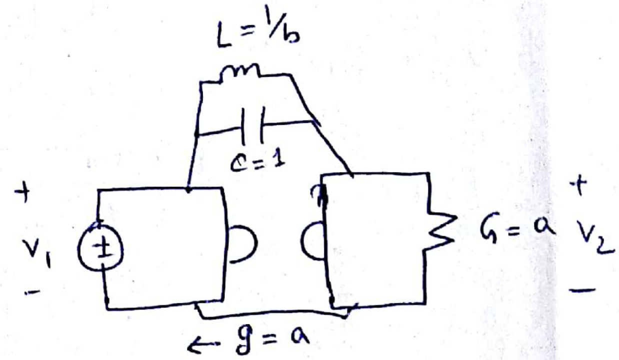
Choosing, $y(s) = sC \Rightarrow \dfrac{V_2}{V_1} = \dfrac{y(s) - G}{y(s) + G}$

So, $$\frac{y - G}{y + G} = \frac{s^2 - as + b}{s^2 + as + b} = \frac{\left(s + \frac{b}{s}\right) - a}{\left(s + \frac{b}{s}\right) + a}$$

equating, $y(s) = s + \dfrac{b}{s}$

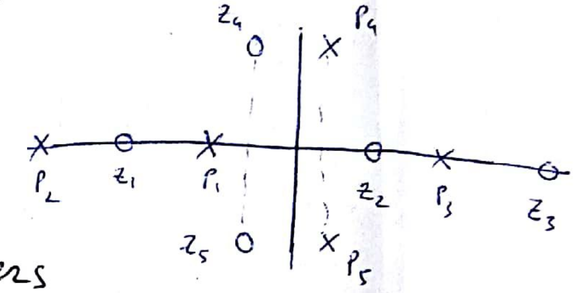
$$G = a$$
$$g = -a$$

So, we can synthesize the circuit as :-



this is an all-pass constant-R circuit utilized via modified Richards' section.

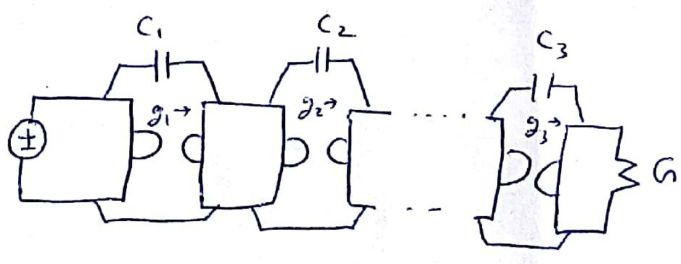
(b) Any rational all-pass $\frac{V_2}{V_1}$ will have the property such that their poles & zeros are equal & opposite from the origin, i.e they are equidistant from the origin, such that their amplitude, $\left|\frac{V_2}{V_1}\right|$ is always unity, & thus they can used as filters for phase angle correction.



i.e, the general form of all all-pass filter is,

$$\frac{V_2}{V_1} = \frac{(s-\alpha_1)(s-\alpha_2)(s-\alpha_3)\cdots(s-\alpha_n)}{(s+\alpha_1)(s+\alpha_2)(s+\alpha_3)\cdots(s+\alpha_n)}$$

we can design this system as a cascade of n-number of Richards' section circuits to get the overall constant-R circuit as:-



where, $G_1 = 1$, & for convenience, choose, $g_1 = g_2 = \cdots = g_n = 1$, & $c_1 = \alpha_1$, $c_2 = \alpha_2$, ..., $c_n = \alpha_n$. Thus, any rational all-pass circuit can be easily synthasized via Richards' sections.

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