ENEE 610 HW #6

1) All poles and zeroes must be in left hafl plane

a) $p_1(s) = s^4 + 8s^3 + 20s^2 + 31s + 30$

$E(s) = s^4 + 20s^2 + 30s^0$

$O(s) = 8s^3 + 31s$

$$\begin{array}{l|l} s^4 + 20s^2 + 30s^0 & 8s^3 + 31s \\ -s^4 - \frac{31}{8}s^2 & \frac{1}{8}s \\ 0 \quad \frac{129}{8}s^2 + 30s^0 & \end{array}$$

This is Hurwitz

$$\begin{array}{l|l} 8s^3 + 31s & \frac{129}{8}s^2 + 30s^0 \\ 0 \quad 16s & \frac{s\,64}{129} \end{array}$$

$$\begin{array}{l|l} 16.1 \quad \frac{129}{8}s^2 + 30s^0 & 16s \\ 0 & 1.006s \end{array}$$

$\boxed{P_1 \text{ is Hurwitz}}$

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b) $s^4 + 6s^3 + 6s^2 + 11s + 30$

$E(s) = s^4 + 6s^2 + 30$

$O(s) = 6s^3 + 11s\ 30$

$$\begin{array}{l|l} s^4 + 6s^2 + 30 & 6s^3 + 11s + 0 \\ s^4 + 2s^2 + & \frac{1}{6}s \\ 4s^2 + 30 & \end{array}$$

$$\begin{array}{l|l} 6s^3 + 11s + 0 & 4s^2 + 30 \\ 0 \quad -35s & \frac{6}{4}s \end{array}$$

$$\begin{array}{l|l} 4s^2 + 30 & -35s \\ & -/ \end{array}$$

$\boxed{\text{Not Hurwitz}}$

c) $s^4 + 7s^3 + 12s^2 + 14s + 20$

$F(s) = s^4 + 12s^2 + 20$

$D(s) = 7s^3 + 14s$

$s^4 + 12s^2 + 20 \;/\; 7s^3 + 14s \;|\; \frac{1}{7}s$ $\quad$ $7s^3 + 14s \;|\; 10s^2 + 20 \;|\; \frac{7}{10}s$

$\cancel{10}$ $\quad$ $0 + 0$

$\boxed{\text{IT IS Hurwitz}}$

2)

$\cancel{a}$ $\quad$ $y(s) = \frac{3s(s^2+4)}{s^2+2}$

$R(s) = \frac{KZ(s) - sZ(K)}{KZ(K) - sZ(s)}$

$Z(s) = \frac{1}{\frac{1}{Z(K)R(s)} + \frac{s}{KZ(K)}} + \frac{1}{\frac{K}{sZ(K)} + \frac{R(s)}{Z(K)}}$

$Z(s) = \frac{1}{y(s)} = \frac{(s^2+2)}{3s\cdot(s^2+4)} = \frac{3}{3s} = \frac{1}{s}$

$R(s) = \frac{\frac{s^2+2}{3s(s^2+4)} - s\cdot\frac{1}{s}}{\frac{1}{s} - \cancel{s}\frac{1}{3}\frac{(s^2+2)}{(s^2+4)}} = \frac{\frac{5s^2+10-3s^2(s^2+4)}{15s(s^2+4)}}{\frac{3(s^2+4)-s(s^2+2)}{15(s^2+4)}}$

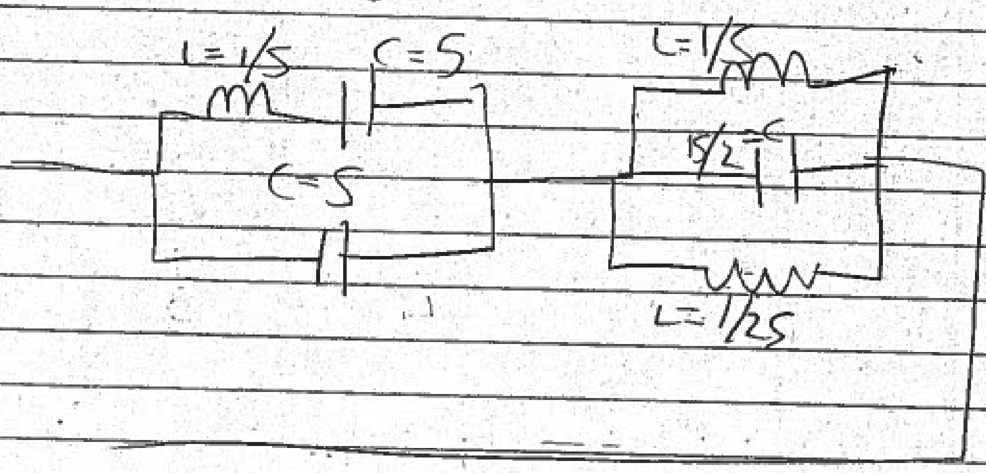
$\frac{5s^2+10-3s^2(s^2+4)}{s(-2s^2+2)} = \frac{-3s^4-7s^2+10}{-10s^2+10}$

$$\frac{3s^4 - 7s^2 + 10}{\cancel{10s^2 + 10}} \qquad - \qquad \frac{3s^4 + 7s^2 + 10}{s(2s^2 + 2)}$$

$3s^4$

$$\frac{\cancel{(s^2+1)}(3s^2+10)}{2s\,\cancel{(s^2+1)}} = \frac{3}{2}s + \frac{5}{8}$$

$$Z(s) = \frac{1}{\frac{1}{\frac{3}{5} + \frac{1}{5}} + \frac{s}{\frac{1}{5}}} + \frac{1}{\frac{1}{\frac{5}{5}} + \frac{15}{2}s + \frac{25}{5}}$$



$K = 2 \qquad Z(s) = \frac{1}{0}$

$P(s)\ \frac{8}{3}\ \frac{2s^2+24}{s(s^2+4)} - s \cdot \frac{1}{8}$

$\frac{1}{9} - 8\ \frac{s^2+2}{38(s^2+4)}$

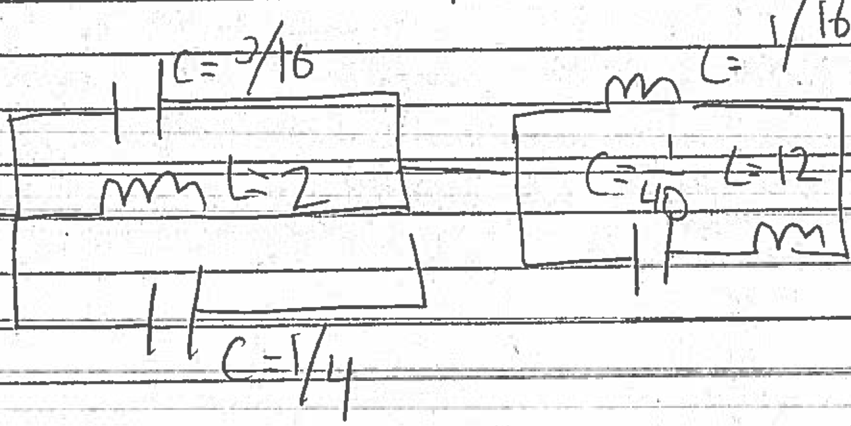
$\frac{16s^2+32-3s^4-12s^2}{25(6s^2+24-8s^2-16)}$

$\frac{3s^4 + 4s^2 - 32}{25(s^2-4)} = \frac{3s^2+8}{25}$

$(s^2-4)(3s^2+8)$

$\frac{3}{2}s + \frac{48}{s}$

$$Z(s) = \cfrac{1}{\cfrac{1}{\frac{3}{16}s + \frac{1}{2s}} + \cfrac{s}{\frac{1}{4}}} + \cfrac{1}{\frac{2}{\frac{s}{8}} + \frac{12s+40}{s}}$$



In order to do $y(s)$, Replace C and L as pictured above and use opposite/different values

$$= \sqrt{a^2+b^2} \angle \tan^{-1}$$

$$Cs^{\alpha} = C(\sigma + jw)^{\alpha} = C(\sqrt{\sigma^2+\sigma^2})^{\alpha} e^{j\alpha \tan^{-1}(w/\sigma)}$$

$$-\pi/2 \le \alpha \tan^{-1}\left(\frac{w}{\sigma}\right) \le \pi/2$$

The ~~real part of~~

$$-\frac{\pi}{2\alpha} \le \tan^{-1}\left(\frac{w}{\sigma}\right) \le \pi/2\alpha$$

so, $\frac{\pi}{2\alpha} < \frac{\pi}{2}$

so, $|\alpha| < 1$