ENEE 610
HW 5

1.) Consider LPR admittance $Y(s) = \dfrac{3s(s^2+4)}{(s^2+1)(s^2+9)}$

a.) Why is this LPR?

$$\frac{3s^3+12s}{s^4+10s^2+9} + \frac{3(-s)^3+12(-s)}{(-s)^4+10(-s)^2+9} = \frac{3s^3+12s-3s^3-12s}{s^4+10s^2+9} = 0$$

$Y(s)+Y(-s)=0$, $n(s)$ & $d(s)$ are real polynomials so lossless

check for PR: no poles/zeros in the right half plane ✓

b. Give state variable realization in companion matrix Y_c, which when loaded on 4 capacitors gives $Y(s)$ at the remaining p

$$i(s) = Y(s)V(s) = (3s^3+12s)\,y_1(s)\,V(s)$$

$$y_1(s) = \frac{1}{s^4+10s^2+9} \qquad V(s) = s^4 i_1 + 10s^2 i_1 + 9 i_1$$

$n_0 = 0$
$n_1 = 2$
$n_2 = 0$
$n_3 =$

$$Y(s) = \frac{3s^3+12s}{s^4+10s^2+9}$$

$$\begin{bmatrix} i \\ \dot{X}_1 \\ \dot{X}_2 \\ \dot{X}_3 \\ \dot{X}_4 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 12 & 0 & 3 \\ 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & -1 \\ -1 & 9 & 0 & 10 & 0 \end{bmatrix} \begin{bmatrix} V \\ X_1 \\ X_2 \\ X_3 \\ X_4 \end{bmatrix}$$

$$\boxed{Y_c = \begin{bmatrix} 0 & 0 & 12 & 0 & 3 \\ 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & -1 \\ -1 & 9 & 0 & 10 & 0 \end{bmatrix}}$$

c on back.

C.) This is a lossless circuit, so we can synthesize with passive components.

2.) Note: for $y = ax^2+bx+c$ $\quad x_1 = \frac{-b+\sqrt{b^2-4ac}}{2a}$

$x_2 = \frac{-b-\sqrt{b^2-4ac}}{2a}$

$x_1 \cdot x_2 = \frac{c}{a}$ $\quad$ for PR BR $\frac{c}{a} > 0$ and $-\frac{b}{a} < 0$

$x_1 + x_2 = -\frac{b}{a}$ $\quad$ for PR: $Re|f(j\omega)| \geq 0$ for BR $|f_1(j\omega)| \leq$

a.) $f_1(s) = \frac{s^2+(2-a)s+1}{s^2+s+a}$ $\quad$ for $n(s)$, $\frac{c}{a} = \frac{1}{1} > 0$ ✓

for $n(s)$, $-\frac{b}{a} = \frac{a-2}{1} < 0$ when $a <$

for $d(s)$, $\frac{c}{a} = \frac{a}{1} > 0$ when $a > 0$

for $d(s)$, $-\frac{b}{a} = -\frac{1}{1} < 0$ ✓

So far could be both PR or BR (PR: $0 < a \leq 2$, BR $0 < a <$

Find $f_1(j\omega) \rightarrow \frac{j^2\omega^2+(2-a)j\omega+1}{j^2\omega^2+j\omega+a} = \frac{-\omega^2+1+(2-a)j\omega}{-\omega^2+a+j\omega}$

$$= \frac{((-\omega^2+1)+(2-a)j\omega)}{((-\omega^2+a)+j\omega)} \frac{((-\omega^2+a)-j\omega)}{((-\omega^2+a)-j\omega)}$$

$$\Rightarrow f_1(j\omega) = \frac{\omega^4-(2a-1)\omega^2+a}{\omega^4-(2a-1)\omega^2+a^2} + \frac{(a-1)\omega(\omega^2-a+1)}{\omega^4-(2a-1)\omega^2+a^2} j$$

$$Re|(f(j\omega))| = \frac{\omega^4-(2a-1)\omega^2+a}{\omega^4-(2a-1)\omega^2+a^2} \geq 0 \quad \begin{cases} \text{for } \omega \in (-\infty,\infty) \\ a > 0 \end{cases}$$

$\therefore$ PR for $\omega \in (-\infty,\infty)$ and $0 < a < 2$

$$|f_1(j\omega)| = \left| \frac{\omega^4-(2a-1)\omega^2+a}{\omega^4-(2a-1)\omega^2+a^2} + j\frac{\omega(a-1)(\omega^2-a+1)}{\omega^4-(2a-1)\omega^2+a^2} \right|$$

for BR $f_1^* f_1 \leq 1$ $\quad$ (used TI-89 for speed)

$(1-2a)\omega^2+a^2 \geq (a^2-4a+2)\omega^2+1$

$$f_1^* f_1 = \frac{\omega^4+(a^2-4a+2)\omega^2+1}{\omega^4+(1-2a)\omega^2+a^2}$$

BR if $a > \frac{\omega^2+1}{\omega^2-1}$ and $0 < a <$

b.) $f_2(s) = \dfrac{(s^2+1)(s^2+4)}{s(s^2+2)(s^2+3)}$

zeros at $-1, -4$

poles at $0, -2$, and -3

can't be BR, but could be PR

$$f_2(j\omega) = \frac{(j^2\omega^2+1)(j^2\omega^2+4)}{j\omega(j^2\omega^2+2)(j^2\omega^2+3)}$$

$$f_2(j\omega) = \frac{(\omega^2-4)(\omega^2-1)}{j\omega(\omega^2-3)(\omega^2-2)} = 0 - j\cdot\frac{(\omega^2-4)(\omega^2-1)}{\omega(\omega^2-3)(\omega^2-2)}$$

$\mathrm{Re}|f_2(j\omega)| = 0 \geq 0 \quad \therefore PR$

c.) $f_3(s) = \dfrac{1}{f(s)}$

rule: if $y(s)$ is PR, then $\frac{1}{y(s)}$ is also PR

f_3 is PR if it follows restrictions given in a.)

...ady prove
...0 and $-\frac{b}{a} < 0$
...oth of these
...omials

$$\frac{s^2+s+a}{s^2+(2-a)s+1} \qquad f(j\omega) = \frac{j^2\omega^2 + j\omega + a}{j^2\omega^2 + (2-a)j\omega + 1}$$

$$f_3(j\omega) = \frac{\omega^4 - (2a-1)\omega^2 + a}{\omega^4 + (a^2-4a+2)\omega^2+1} - j\,\frac{(a-1)\omega(\omega^2-a+1)}{\omega^4+(a^2-4a+2)\omega^2+1}$$

for BR $|f_3^* f_3| \leq 1$ $\quad f_3^* f_3 = \dfrac{\omega^4 + (1-2a)\omega^2 + a^2}{\omega^4 + (a^2-4a+2)\omega^2+1}$ $\quad$ we already know $0 < a < 2$

for BR $(a^2-4a+2)\omega^2 + 1 > (1-2a)\omega^2 + a^2$

d.) $f_4(s) = \dfrac{s^2 - as + 4}{s^2 + as + 4}$ for $n(s)$: $\frac{c}{a} = \frac{4}{1} > 0$ ✓

$-\frac{b}{a} = \frac{a}{1} < 0$ when $a <$

for $d(s)$: $\frac{c}{a} = \frac{4}{1} > 0$ ✓

$-\frac{b}{a} = \frac{-a}{1} < 0$ when $a > 0$

There is no a that prevents both poles + zeroes from appearing in the right half plane

This is neither PR nor BR

3. Partial Fraction Expansion for tan(s) + tanh(s) are:

$$\tan(s) = \sum_{n=1}^{\infty} \frac{8s}{(2n-1)^2\pi^2 - 4s^2} = -8s \sum \frac{1}{4s^2 - (2n-1)^2\pi^2}$$

zero at $s=$
pole at $s=$

$$\tanh(s) = \sum_{n=1}^{\infty} \frac{8s}{(2n-1)^2\pi^2 + 4s^2} = 8s \sum_{n=1}^{\infty} \frac{1}{4s^2 + (2n-1)^2\pi^2}$$

zero at $s=0$
pole at $s=$
$s = j$

a.) Discuss why 1 is PR and the other is not

tan(s) is not PR because it has a pole in the right half

tanh(s) is PR because it has no poles or zeroes in the RH

b.) The infinite product Expansions for sin + cos are

$$\sin(\pi s) = \pi s \prod_{n=1}^{\infty} \left(1 - (s/n)^2\right)$$

$$\cos(\pi s) = \prod_{n=1}^{\infty} \left(1 - (2s/\{2n-1\})^2\right)$$

Form infinite product expansion for tan(s) + infer the infinite product expansion for tanh(s) = sin

$$\tan(\pi s) = \frac{\sin(\pi s)}{\cos(\pi s)} = \pi s \prod_{n=1}^{\infty} \frac{1-\left(\frac{s}{n}\right)^2}{1-\left(\frac{2s}{(2n-1)}\right)^2}$$

$$\boxed{\begin{aligned} \tan(s) &= s \prod \frac{1-\left(\frac{s}{n}\right)^2}{1-\left(\frac{2s}{2n-1}\right)^2} \\ \tanh(s) &= -j \prod \frac{1-\left(\frac{js}{n}\right)^2}{1-\left(\frac{2js}{2n-1}\right)^2} \end{aligned}}$$

$$\tanh(x) = \frac{\sinh(x)}{\cosh(x)} = \frac{-j\sin}{\cos}$$