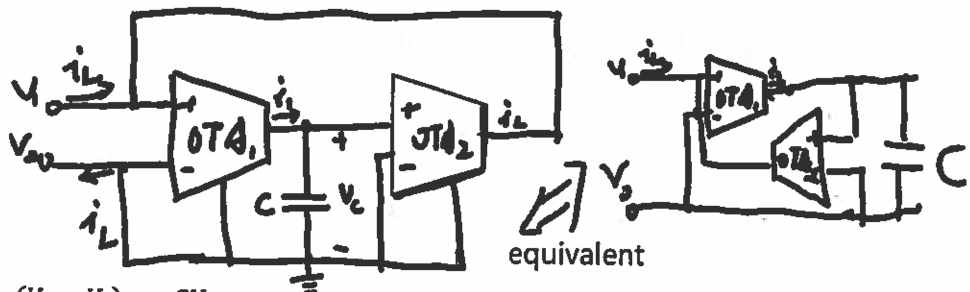
Hw_4

2018年10月3日 10:29

1. (40 points, negative & positive C's,L's)
   Given that a capacitor, of capacitance C>0, and four OTAs are available (of gains g_{mi} for i=1, ...,4). Assume that the g_{mi}>0 with signs determined by the OTA input connections and use these to
   a) Draw a circuit for a positive inductor giving inductance value L; give the value of L in terms of C and the different g_{mi} used.
   b) Draw a circuit for an inductor giving a negative inductance value –L, where L is the value obtained in part a).
   c) Draw a circuit to give a capacitor of negative capacitance and give the value of this capacitor.
   d) Comment upon where gyrators can be used in any of the above.

a)



$i_1 = g_{m1}(V_1 - V_2) = sCV_C$

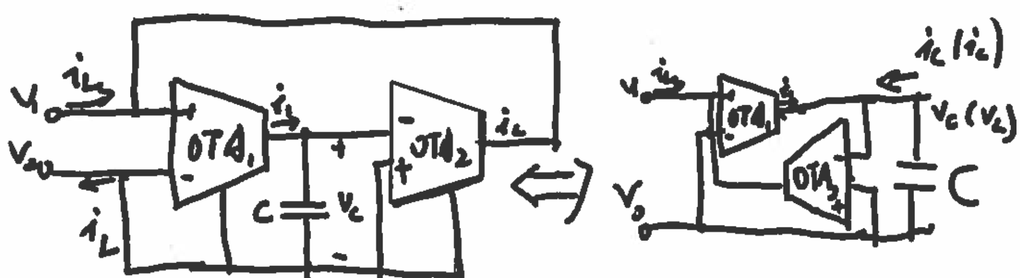
$\Rightarrow V_C = \dfrac{g_{m1}(V_1 - V_2)}{sC}$

$i_L = g_{m2}V_C = \dfrac{g_{m1}g_{m2}(V_1 - V_2)}{sC}$

the impedance of this circuit, which is also the equivalent inductance, is:

$Z = L_{eq} = \dfrac{V_1 - V_2}{i_L} = \dfrac{sC}{g_{m1}g_{m2}}$

b)



Problem b) is relatively similar to problem a).
We just have to inverse the second OTA and would get the negative inductance.

$i_1 = g_{m1}(V_1 - V_2) = sCV_C$

$\Rightarrow V_C = \dfrac{g_{m1}(V_1 - V_2)}{sC}$

$i_L = -g_{m2}V_C = -\dfrac{g_{m1}g_{m2}(V_1 - V_2)}{sC}$

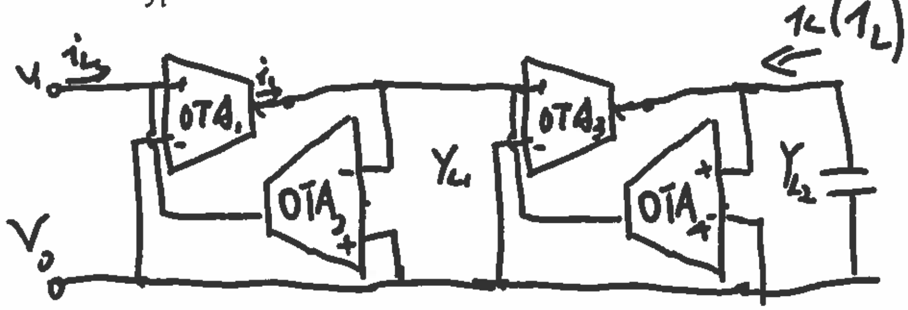
the impedance of this circuit, which is also the equivalent inductance, is:

$V_1 - V_2$ sC

$$Z = L_{eq} = \frac{V_1 - V_2}{i_L} = -\frac{sC}{g_{m1}g_{m2}}$$

c)
So basically the relationship for Y_{in} and Y_L of the above circuit is:

$$y_{in} = \frac{-g_{m_{12}}g_{m_{21}}}{y_l} \text{ (the minus sign depends on the direction of the OTAs)}$$



$$y_{in} = \frac{-g_{m_1}g_{m_2}}{y_{L_1}}, \text{where } y_{L_1} = \frac{g_{m_3}g_{m_4}}{y_{L_2}}$$

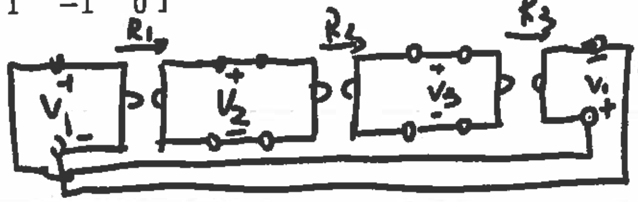
$$y_{in} = \frac{-g_{m_1}g_{m_2}}{g_{m_3}g_{m_4}} y_{L_2} = \frac{-g_{m_1}g_{m_2}}{g_{m_3}g_{m_4}} C \text{ where } C_{eq} = \frac{g_{m_1}g_{m_2}}{g_{m_3}g_{m_4}}$$

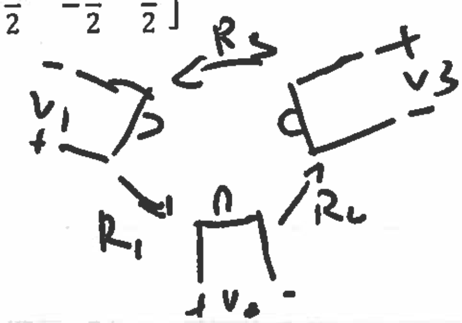
d)
for (a) when $g_{m_1} = -g_{m_2} = g_m$, *we can replace it with a gyrator.*
for (b) we can't not use gyrator
for (c) when $g_{m_1} = -g_{m_2} = g_m$, *we can replace it with a gyrator.*

2. (40 points, circulators)
   a) For the three port circulator used in class give its admittance matrix and from that draw a circuit using gyrators to realize its scattering matrix.
   b) Show that $Y=-Y^T$ and investigate the total instantaneous power in, $p(t)=v(t)^T i(t)$.
   c) Give also its impedance matrix and compare with the Y of part a).

$$a)\ Y = (s + 1_2)^{-1}(1_2 - s) = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix}$$





$$b)\ -Y^T = -\begin{bmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix}^T = \begin{bmatrix} 0 & -1 & 1 \\ 1 & 0 & -1 \\ -1 & 1 & 0 \end{bmatrix}^T = \begin{bmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix} = Y$$

this is a lossless circuit, it is passive circuit.

$p(t) = v(t)^T i(t) = 0$

$c)\ Z = Y^{-1}, however, \det(Y) = 0, thus\ Y^{-1}\ does\ not\ exist.$

3. (20 points, multiport circulator and use)
   A 3n-port circulator is obtained by replacing each 1 in the 3-port circulator by 1n, the nxn identity, in the 3n-port device.
   a) Give the 3n-port circulator scattering matrix, S_{3n}.
   b) Load the second set of n ports in an n-port of scattering matrix S_a and the last n ports in S_b. Give the resulting input scattering matrix S_{in} seen at the first n-ports.
   c) Showing all the ports, draw a schematic diagram for the connection of part b) when n=2. For this a two-level 3D drawing with odd circulator ports on one level and even on another may be convenient.

a)

$$S_{3n} = \begin{bmatrix} 0 & 0 & 1_n \\ 1_n & 0 & 0 \\ 0 & 1_n & 0 \end{bmatrix}$$

b)

$v_2^i = reflected\ from\ load\ N_b = S_a . v_2^r, where\ S_a = [1_n]$

$v_3^r = v_2^i = into\ load\ on\ port\ 3 = N_a$

$v_3^i = reflected\ voltage\ from\ load\ N_b = S_b v_2^r, where\ S_b = [1_n]$

$S_{in} = [1_n] = S_a * S_b$

c)