HW 3

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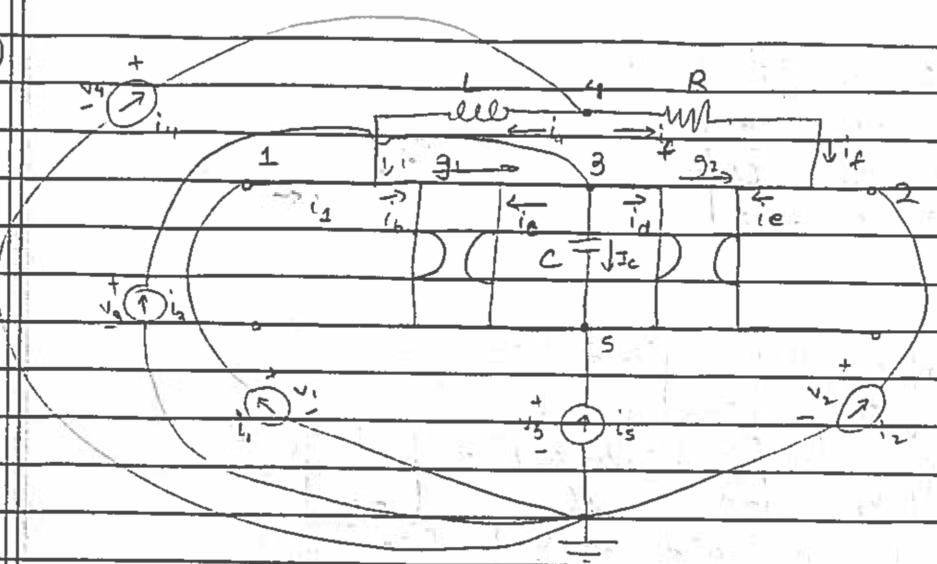
(1)



a) $i_1 = i_b - i_a = g_1(V_3 - V_5) - \frac{1}{sL}(V_4 - V_1) = \frac{1}{sL}V_1 + g_1 V_3 - \frac{1}{sL}V_4 - g_1 V_5.$

$i_2 = i_e - i_f = -g_2(V_3 - V_5) - g(V_4 - V_2) = gV_2 - g_2 V_3 - gV_4 + g_2 V_5$

$i_3 = i_c + i_d + I_c = -g_1(V_1 - V_5) + g_2(V_2 - V_5) + sC(V_3 - V_5) = -g_1 V_1 + g_2 V_2 + sC V_3 + (g_1 - g_2 - sC) V$

$i_4 = i_a + i_f = \frac{1}{sL}(V_4 - V_1) + g(V_4 - V_2) = -\frac{1}{sL}V_1 - gV_2 + (g + \frac{1}{sL})V_4$

$i_5 = -(i_c + i_d + I_c) = g_1(V_1 - V_5) - g_2(V_2 - V_5) - sC(V_3 - V_5)$

$= g_1 V_1 - g_2 V_2 - sC V_3 + (g_2 + sC - g_1) V_5.$

Then,

$$Y_{ind} = \begin{bmatrix} \frac{1}{sL} & 0 & g_1 & -\frac{1}{sL} & -g_1 \\ 0 & g & -g_2 & -g & g_2 \\ -g & g_2 & sC & 0 & g_1 - g_2 - sC \\ -\frac{1}{sL} & -g & 0 & g + \frac{1}{sL} & 0 \\ g_1 & -g_2 & -sC & 0 & g_2 + sC - g_1 \end{bmatrix}$$

b) Eliminate $3 \Rightarrow i_3 = 0.$

$$Y_{def} = \begin{bmatrix} \frac{1}{sL} & 0 & g_1 & -\frac{1}{sL} \\ 0 & g & -g_2 & -g \\ -g & g_2 & sC & 0 \\ -\frac{1}{sL} & -g & 0 & g + \frac{1}{sL} \end{bmatrix}$$

$-g_1 V_1 + g_2 V_2 + sC V_3 = 0.$

$\therefore V_3 = \dfrac{g_1 V_1 - g_2 V_2}{sC}.$

Then,

$$\begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{bmatrix} = \begin{bmatrix} \frac{1}{sL} & 0 & g_1 & -\frac{1}{sL} \\ 0 & g & -g_2 & -g \\ -g & g_2 & sC & 0 \\ -\frac{1}{sL} & -g & 0 & g+\frac{1}{sL} \end{bmatrix}$$

$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} \frac{1}{sL} & 0 \\ 0 & g \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} + \begin{bmatrix} g_1 \\ -g_2 \end{bmatrix} V_3 + \begin{bmatrix} -\frac{1}{sL} \\ -g \end{bmatrix} V_4$$

$$= \begin{bmatrix} \frac{1}{sL} & 0 \\ 0 & g \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} + \begin{bmatrix} g_1 \\ -g_2 \end{bmatrix} \frac{g_1 V_1 - g_2 V_2}{sC} + \begin{bmatrix} -\frac{1}{sL} \\ -g \end{bmatrix} V_4$$

$$= \begin{bmatrix} \frac{1}{sL} & 0 \\ 0 & g \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} + \frac{1}{sC} \begin{bmatrix} g_1^2 V_1 - g_1 g_2 V_2 \\ -g_1 g_2 V_1 + g_2^2 V_2 \end{bmatrix} + \begin{bmatrix} -\frac{1}{sL} \\ -g \end{bmatrix} V_4$$

$$= \begin{bmatrix} \frac{1}{sL} & 0 \\ 0 & g \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} + \frac{1}{sC} \begin{bmatrix} g_1^2 & -g_1 g_2 \\ -g_1 g_2 & g_2^2 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} + \begin{bmatrix} -\frac{1}{sL} \\ -g \end{bmatrix} V_4$$

$$\therefore V_{2port} = \frac{1}{sC} \begin{bmatrix} \frac{C}{L} g_1^2 & -g_1 g_2 \\ -g_1 g_2 & sCg \end{bmatrix}$$

<u>Eliminate 4</u> $\Rightarrow$ $i_4 = 0$

$$-\frac{1}{sL} v_1 - g V_2 + \left(g + \frac{1}{sL}\right) V_4 = 0$$

or, $$V_4 = \frac{\frac{1}{sL} V_1 + g V_2}{g + \frac{1}{sL}}$$

$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} \frac{1}{sL} & 0 \\ 0 & g \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} + \begin{bmatrix} -\frac{1}{sL} \\ -g \end{bmatrix} \frac{\frac{1}{sL} V_1 + g V_2}{g + \frac{1}{sL}} + \begin{bmatrix} g_1 \\ -g_2 \end{bmatrix} V_3$$

$$= \begin{bmatrix} \frac{1}{sL} & 0 \\ 0 & g \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} + \frac{1}{g+sL} \begin{bmatrix} -\frac{1}{s^2L^2} V_1 - \frac{g}{sL} V_2 \\ -\frac{g}{sL} V_1 - g^2 V_2 \end{bmatrix} + \begin{bmatrix} g_1 \\ -g_2 \end{bmatrix} V_3$$

$$= \begin{bmatrix} \frac{1}{sL} & 0 \\ 0 & g \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} + \begin{bmatrix} -\frac{1}{s^2L^2(g+sL)} & -\frac{g}{sL(g+sL)} \\ -\frac{g}{sL(g+sL)} & -\frac{g^2}{g+sL} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} + \begin{bmatrix} g_1 \\ -g_2 \end{bmatrix} V_3$$

$$\therefore V_{2port} = \frac{1}{g+sL} \begin{bmatrix} \frac{g+sL}{sL} - \frac{1}{s^2L^2} & -\frac{g}{sL} \\ -\frac{g}{sL} & g(g+sL) - g^2 \end{bmatrix}$$

② ⓐ $T(s) = B^T(sE - A)^{-1}B.$

$$(sE - A) = s\begin{bmatrix} k1 & 1 \\ -1 & k2 \end{bmatrix} - \begin{bmatrix} -h1 & b1 \\ -a1 & -h2 \end{bmatrix}$$

$$= \begin{bmatrix} sk1+h1 & s-b1 \\ a1-s & sk2+h2 \end{bmatrix}$$

$$(sE - A)^{-1} = \frac{1}{(sk1+h1)(sk2+h2)-(s-b1)(a1-s)}\begin{bmatrix} sk2+h2 & b1-s \\ s-a1 & sk1+h1 \end{bmatrix}$$

$$T(s) = \frac{1}{(sk1+h1)(sk2+h2)-(s-b1)(a1-s)}\begin{bmatrix} 1 & 0 \end{bmatrix}\begin{bmatrix} sk2+h2 & b1-s \\ s-a1 & sk1+h1 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$= \frac{1}{(sk1+h1)(sk2+h2)-(s-b1)(a1-s)}\begin{bmatrix} 1 & 0 \end{bmatrix}\begin{bmatrix} sk2+h2 \\ s-a1 \end{bmatrix}$$

$$= \frac{1}{(sk1+h1)(sk2+h2)-(s-b1)(a1-s} \times s-a1$$

$$= \frac{1}{s-b1 + \frac{(sk1+h1)(sk2+h2)}{s-a1}}$$

$$= \frac{1}{s-b1 + \frac{(s+h1/k1)(s+h2/k2)}{\frac{s}{k1k2} - \frac{a1}{k1k2}}}.$$

$\therefore a = 1,\ b = b1,\ c = \frac{1}{k_1k_2},\ d = -\frac{a1}{k1k2},\ e = \frac{-h1}{k1},\ f = \frac{-h2}{k2}.$

ⓑ The above equation gives state space realization

ⓒ $z(s) = \frac{2s+6}{(s+2)(s+6)}$

$A_1(s) = (s+2)(s+6) = s^2+8s+12$

$A_2(s) = 2s+6$

b2) Eliminate 3:

$$V_3 = \frac{g_1 V_1 - g_2 V_2}{sC}$$

$$V_4 = \frac{\frac{1}{sL} V_1 + g V_2}{g + \frac{1}{sL}}$$

$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} \frac{1}{sL} & 0 \\ 0 & g \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} + \frac{1}{g+sL} \begin{bmatrix} -\frac{1}{s^2L^2} & \frac{-g}{sL} \\ \frac{-g}{sL} & -g^2 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} + \frac{1}{sC} \begin{bmatrix} g_1^2 & -g_1 g_2 \\ -g_1 g_2 & g_2^2 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$= \frac{1}{sC(g+sL)} \begin{bmatrix} \frac{C(g+sL)}{L} & 0 \\ 0 & sCg(g+sL) \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} + \frac{1}{g+sL} \begin{bmatrix} -\frac{1}{s^2L^2} & \frac{-g}{sL} \\ \frac{-g}{sL} & -g^2 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} + \frac{1}{sC} \begin{bmatrix} g_1^2 & -g_1 g_2 \\ -g_1 g_2 & g_2^2 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$\therefore Y_{2port} = \frac{1}{sC(g+sL)} \begin{bmatrix} \frac{C(g+sL)}{L} - \frac{C}{sL^2} + g_1^2(g+sL) & \frac{-gC}{sL} - g_1 g_2 (g+sL) \\ -\frac{gC}{L} & -g^2 sC + g_2^2(g+sL) \end{bmatrix}.$$

(c) $\therefore$ Y's differ.