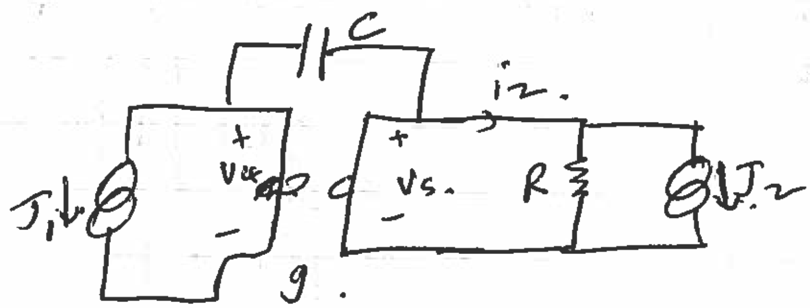
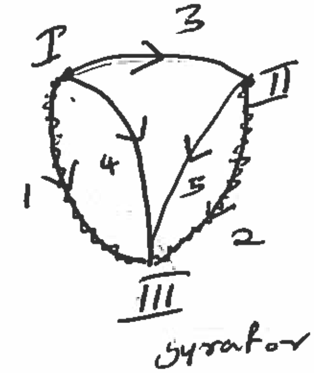
Chanaka Singhabahu.

Homework 2.

① (a)





(b) $Av = Bi$

$$i = \begin{bmatrix} 0 & G \\ -G & 0 \end{bmatrix} v.$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & G_R & 0 & 0 & 0 \\ 0 & 0 & sC & 0 & 0 \\ 0 & 0 & 0 & 0 & G \\ 0 & 0 & 0 & -G & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \\ i_6 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & -1 & 0 & 1 \end{bmatrix} \qquad J_s = \begin{bmatrix} J_1 \\ J_2 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$-CJ_s = -\begin{bmatrix} 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} J_1 \\ J_2 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} -J_1 \\ -J_2 \end{bmatrix}$$

$$CAC^T = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & G_R & 0 & 0 & 0 \\ 0 & 0 & sC & 0 & 0 \\ 0 & 0 & 0 & 0 & G \\ 0 & 0 & 0 & -G & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & -1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & sC & 0 & G \\ 0 & G_R & -sC & -G & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & -1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} sC & -sC+G \\ -sC-G & G_R+sC \end{bmatrix}$$

$$CAC^T V_t = -CJ_s$$

$$\begin{bmatrix} sC & -sC+G \\ -sC-G & G_R+sC \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} -J_1 \\ -J_2 \end{bmatrix}$$

$$\det(CAC^T) = sC(G_R+sC) + (G-sC)(G+sC)$$

$$= sC(G_R \cancel{+sC}) + \cancel{G(G+sC)}\, G^2$$

$$= sC(G_R \cancel{+sC}) + G^2$$

$$= G_R sC + G^2$$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix}_b = \frac{-1}{sC(G_R) + G^2} \begin{bmatrix} G_R + sC & sC - G \\ sC + G & sC \end{bmatrix} \begin{bmatrix} +J_1 \\ +J_2 \end{bmatrix}.$$

(c).

Impedance Matrix

$$Z = \frac{-1}{sC(G_R) + G^2} \cdot \begin{bmatrix} G_R + sC & sC - G \\ sC + G & sC \end{bmatrix}$$

Admittance

$$Y = Z^{-1}$$

$$= -\begin{bmatrix} sC & -sC + G \\ -sC - G & G_R + sC \end{bmatrix}$$

(2) $z(s) = \dfrac{2s+6}{(s+2)(s+6)} = \dfrac{2s+6}{s^2+8s+12}.$

$T(\infty) = \dfrac{2s}{s^2} \to 0. \quad \therefore D = 0.$

$A = \begin{bmatrix} 0 & 1 \\ -12 & -8 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad C = \begin{bmatrix} 6 & 2 \end{bmatrix}$

$D = 0.$

$y = \left[C (sI_2 - A)^{-1} B + D \right] u.$

$z = \begin{bmatrix} 6 & 2 \end{bmatrix} \begin{bmatrix} s & -1 \\ 12 & s+8 \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$z(s) = \dfrac{2s+6}{s^2+8s+12}.$

$\therefore A = \begin{bmatrix} 0 & 1 \\ -12 & -8 \end{bmatrix}$

$B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$C = \begin{bmatrix} 6 & 2 \end{bmatrix}$

$D = 0.$

(b) $\cancel{A = \begin{bmatrix} -6 & 0 \\ 0 & -2 \end{bmatrix}}$ $A = \begin{bmatrix} -2 & 0 \\ 0 & -6 \end{bmatrix}$

$\cancel{B = [1\ 2]}$ $B = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ $C = \frac{1}{2}[1\ 3]$

$s \to \infty$. $\frac{2s}{s^2} = 0$. $D = 0$.

$$z(s) = \frac{1}{2}[1\ 3]\begin{bmatrix} s+2 & 0 \\ 0 & s+6 \end{bmatrix}^{-1}\begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$= \frac{1}{2}\frac{[1\ 3]}{(s+6)(s+2)}\begin{bmatrix} s+6 & 0 \\ 0 & s+2 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$= \frac{2s+6}{(s+6)(s+2)} = \frac{2s+6}{s^2+8s+12}$$

(c) $y(s) = \frac{1}{z(s)} = \frac{s^2+8s+12}{2s+6}$

$y = C(sI_2 - A)^{-1}B + D$.

$s \to \infty$. $\frac{s^2}{2s} \to \infty$. $D \to \infty$.

It cannot be realized.