ENEE 610
Th 12/06/18

Note Title 12/6/2018

$\dot{x} = Ax + Bu, \quad y = Cx + Du, \quad x = k\text{-vector}$ linear time-invariant

solution
$$e^{At} = \sum_{i=0}^{\infty} \frac{(At)^i}{i!} = 1_k + At + \frac{1}{2}A^2t^2 + \dots, \quad \frac{de^{At}}{dt} = Ae^{At}$$

$\therefore$ if $u = 0$, $\dot{x} = Ax$ is solved, $x(t) = e^{At}x(0)$, $t > 0$

if $u \neq 0$ then $x(t) = e^{At}x(0) + \left(\int_0^t e^{A(t-\tau)}Bu(\tau)\,d\tau\right)$ $t \geq 0$

To solve $E\dot{x} = Ax + Bu$, need the Drazin inverse

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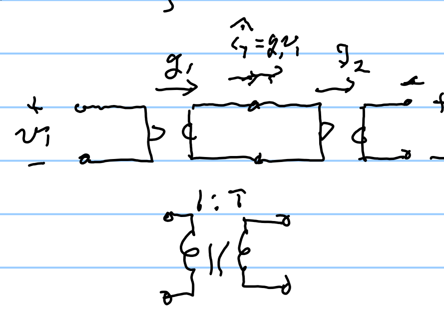
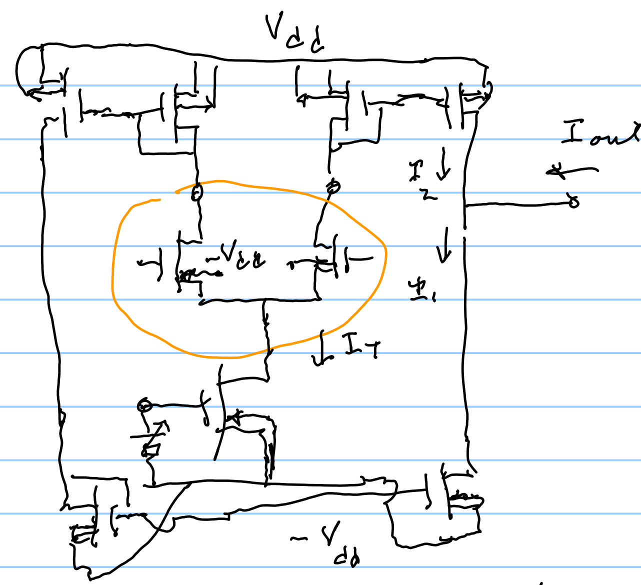
Transformer

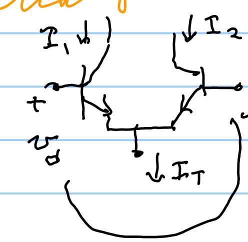


$v_2 = \frac{g_1}{g_2}v_1 = Tv_1$ $\quad T =$ turns ratio



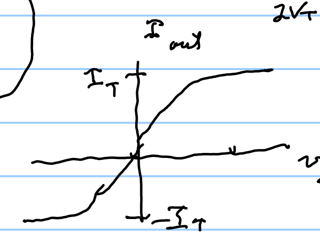
= differential pair

if BJT



$I_{out} = I_1 - I_2$

$= I_T \tanh\left(\frac{v_d}{2V_T}\right)$



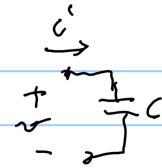
small signal gain is

$$\left.\frac{dI_{out}}{dv_d}\right|_{v_d=0} \approx \left.\frac{d\, I\tanh(v_d/2V_T)}{dv_d}\right|_{v_d=0} \approx \left. I\left(1-\tanh^2\left(\frac{v_d}{2V_T}\right)\right)\cdot\frac{1}{2V_T}\right|_{v_d=0} = I_T \cdot \frac{1}{2V_T}$$

v_d small signal gain

V_T = thermal voltage

$= \frac{kT}{|q|} \approx 26\,mV$ @ room Temp initial conditions:



$i = C \frac{dv}{dt}$

$v(0_-) = v(0_+)$ if we switch the capacitor & no ∞ current is allowed

if C is time varying: $i = \frac{dq}{dt} = \frac{d\,C \cdot v}{dt} = \partial C \cdot v = C \cdot \frac{dv}{dt} + \frac{dC}{dt} \cdot v$

q = charge on capacitor $= C \cdot v$

if C is voltage dependent & time dependent $\underbrace{\frac{d\,C(v,t)\,v}{dt}}$ $= \underbrace{\frac{\partial C(v,t)}{\partial v}} \cdot v + \frac{\partial C(v,t)}{\partial t} \cdot v + C \cdot \frac{dv}{dt}$