ENEE 610
Tu 12/04/18

Note Title 11/16/2018

$s = \sigma + j\omega$

strictly Hurwitz

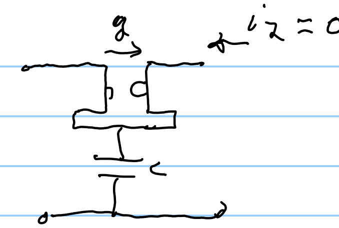
$\sigma < 0$

stable if simple } Hurwitz

$\sigma = 0$ unstable if multiple poles

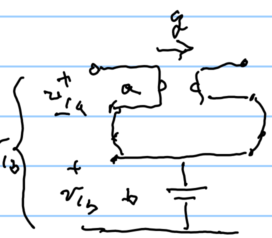
RHP : unstable

pole positions of rational transfer functions real or complex coefficients sensitivity example

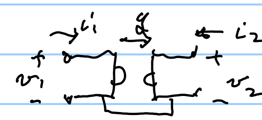


$\left.\frac{v_o}{v_i}\right|$ open circuit load, $i_2 = 0$

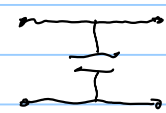
$Y = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix}$



$\Rightarrow Z = Z_a + Z_b$



$Y_a = \begin{bmatrix} 0 & g \\ -g & 0 \end{bmatrix}$, $Z_a = Y_a^{-1} = \frac{1}{g^2}\begin{bmatrix} 0 & -g \\ g & 0 \end{bmatrix} = \begin{bmatrix} 0 & -1/g \\ 1/g & 0 \end{bmatrix}$



$Z_b = \begin{bmatrix} \frac{1}{ac} & 1/ac \\ \frac{1}{ac} & \frac{1}{ac} \end{bmatrix}$

$Z = \begin{bmatrix} \frac{1}{ac}+0 & \frac{1}{ac} - 1/g \\ \frac{1}{ac}+\frac{1}{g} & \frac{1}{ac} \end{bmatrix}$; $\Delta Z = \det Z = \left(\frac{1}{ac}\right)^2 - \left(\left(\frac{1}{ac}\right)^2 - \frac{1}{g^2}\right) = \frac{1}{g^2}$ $Y = \frac{1}{1/g^2}\begin{bmatrix} 1/ac & \frac{-1}{ac}+\frac{1}{g} \\ \frac{-1}{ac}-\frac{1}{g} & 1/ac \end{bmatrix}$

$i_2 = y_{21}v_1 + y_{22}v_2 = 0$ in original, $\frac{v_2}{v_1} = \frac{-y_{21}}{y_{22}} = \frac{-\left(\frac{-1}{ac}-\frac{1}{g}\right)}{1/ac} = 1 + \frac{ac}{g}$

sensitivity $S_x^T = \frac{1}{T}\frac{dT}{dx} \Rightarrow S_C^{v_2/v_1} = \frac{1}{(1+\frac{sC}{g})}\cdot\frac{d(1+\frac{sC}{g})}{dC} = \frac{s/g}{1+\frac{sC}{g}} = \frac{s}{g+sC} =$

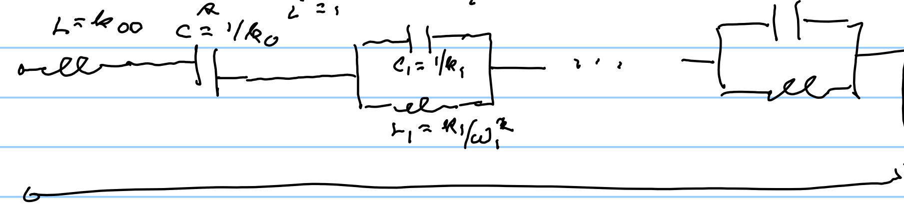
if $s = j\omega \Rightarrow S_x^{T(j\omega)} = \frac{1}{|T|e^{j\angle T}}\cdot\frac{d|T|e^{j\angle T}}{dx} = \frac{d|T|}{dx}\cdot\frac{e^{j\angle T}}{|T|\cdot e^{j\angle T}} + \frac{|T|\cdot\frac{de^{j\angle T}}{dx}}{|T|e^{j\angle T}} = \frac{1}{|T|}\frac{d|T|}{dx} + \frac{e^{j\angle}}{e^{j\angle}}\frac{d\angle T}{dx}\cdot j$

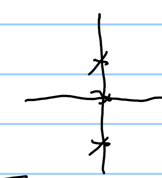
$= S_x^{|T|} + j\frac{d\angle T}{dx}$

RC synthesis

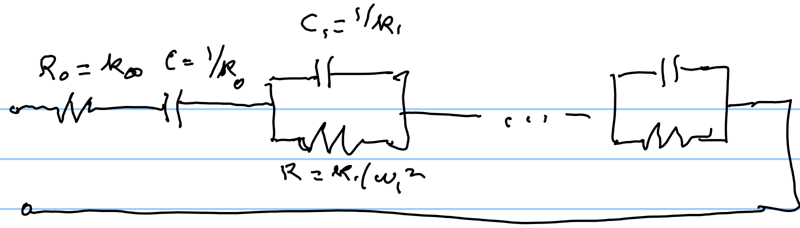
1st Foster, LC $\quad Z(s) = k_\infty \cdot s + \frac{k_0}{s} + \sum_{i=1}^{i=n}\frac{k_i s}{s^2+\omega_i^2}$

LC $\quad L = k_\infty \quad C = 1/k_0$



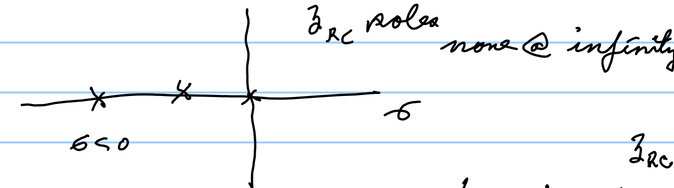


Replace each L by an R to get an RC circuit



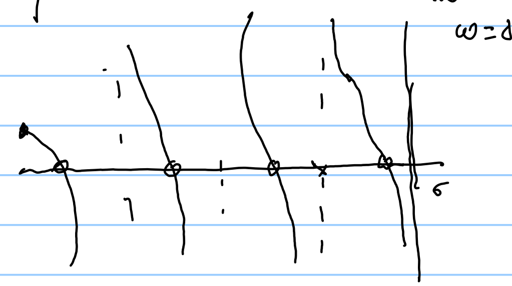
$$\Rightarrow z_{RC} = R_\infty + \frac{k_0}{s} + \sum_{i=1}^{n} \frac{k_i}{s+\omega_i^2}, \quad \text{all } k_s^i \geq 0, \ \omega_i^2 \geq 0$$

$$z_1 = \frac{1}{\frac{s}{k_1} + \frac{\omega_1^2}{k_1}} = \frac{k_1}{s+\omega_1^2}$$



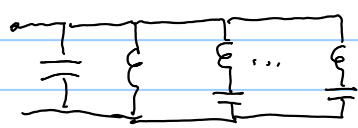
look @ $\left.\frac{dz_{RC}}{d\sigma}\right|_{\omega=0} = -\frac{k_0}{\sigma^2} + \sum_{i=1}^{\infty} \frac{-k_i}{(\sigma+\omega_i^2)^2}$

derivative is always < 0

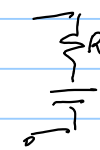


$y_{RC} = \frac{1}{z_{RC}}$ = look @ 2nd Foster

$$y_{LC} = \hat{k}_\infty s + \frac{\hat{k}_0}{s} + \sum_{i=1}^{\hat{n}} \frac{\hat{k}_i s}{s^2 + \hat{\omega}_i^2}$$



$z_{i_{LC}} = sL + \frac{1}{sC} \Rightarrow$



$$\Rightarrow z_{i_{RC}} = R + \frac{1}{sC} = \frac{sCR+1}{sC} = \frac{s + G/C}{sG} = \frac{s+G/C}{sG}$$

$$y_{RC} = \hat{k}_\infty s + \hat{k}_0 + \sum_{i=1}^{\hat{n}} \frac{sC_i}{sC_iR_i+1}$$ is not a partial fraction

form $\frac{y_{RC}}{s} = \hat{k}_\infty + \frac{\hat{k}_0}{s} + \sum_{i=1}^{\hat{n}} \frac{G_i}{s+G_i/C_i}$ This is a partial fraction expansion of $\frac{y_{RC}}{s}$

∴ for 2nd Foster make a partial fraction expansion of Y/s & then synthesize each term in Y after multiply back the s.

for 1st Cauer as we expand about $s = \infty$; remove poles of y & constants of Z

for 2nd Cauer we expand about $s=0$; remove constants @ $s=0$ of Y, for Z, poles @ 0

Ex: $Y(s) = \frac{s(s+2)}{(s+1)} \underset{\text{2nd Foster}}{\Longrightarrow} Y/s = \frac{s+2}{s+1} = 1 + \frac{k_1}{s+1}$ for k_1: $s+2 = \cancel{s}+1+k_1 \Rightarrow k_1 = 1$

$y = s + \frac{s}{s+1} \Rightarrow$