Note Title 11/15/2018

ENEE 610
11/15/18 + 11/16/18

adjoint for sensitivity $S_x^T = \frac{x}{T}\frac{dT}{dx}$; Y^a x = a parameter; T = transfer function

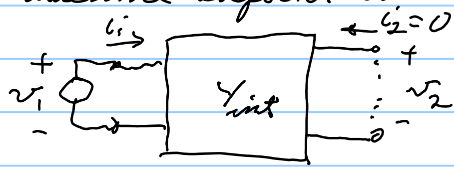
Y^a = matrix adjoint, $Y^a = Y^T$

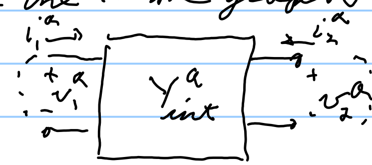
$\underbrace{v^T i}_{\text{scalar product}} = \langle v, i\rangle = v^T Y v = (v^T Y v)^T = v^T Y^T v$

$= \langle v^a, i^a \rangle \Rightarrow Y^T = Y^a$

$= \langle v^a, Y^a v^a \rangle$

assume adjoint circuit has the same graph as original circuit





$i_b^T v_b = 0$; $i_b^T v_b^a = 0$ b = branch

$v_b^T i_b = 0$, $i_b^a v_b = 0$

$\therefore\ i_b^T v_b^a - i_b^a v_b = 0 \Rightarrow -\left[i_1 v_1^a + i_2 v_2^a - i_1^a v_1 - i_2^a v_2\right] + i_{int}^T v_{int}^a - i_{int}^{aT} \cdot v_{int} = 0$ ← differentiate

minus due to external branch polarities

here v_2/v_1 = transfer function of interest, x = parameter {ex: C, g, temp, anything that varies}

assume Y_{int} changes with x but Y_{int}^a does not

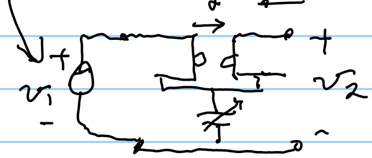
$i_{int}^T v_{int}^a = v_{int}^T \cdot Y^T v_{int}^a$

$i_{int}^{aT} \cdot v_{int} = v_{int}^{aT} Y^{aT} v_{int}$

use Y for internal branch by branch Y_{int}

fixed applied voltage choose = 1v (ok as linear) circuit

$i_2 \equiv 0$ as open circuit

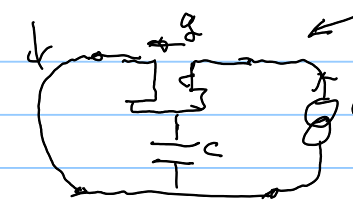


$\Longrightarrow$ adjoint

$v_1^a \equiv 0$

fixed circuit no changes with x

$\frac{d i_2^a}{dx} \equiv 0$



$\Leftarrow$ choose $= -1$

desired term as has $dv_2/dx = \frac{d\, v_2/v_1}{dx}\Big|_{v_1=1}$

above eq.

$$\frac{d}{dx} = 0 = -\left[\frac{di_1}{dx} \cdot v_1^a + i_1 \frac{dv_1^a}{dx} + \frac{di_2}{dx} v_2^a + i_2 \frac{dv_2^a}{dx} - \frac{di_1^a}{dx} v_1 - i_1^a \frac{dv_1}{dx} - \frac{di_2^a}{dx} v_2 - i_2^a \frac{dv_2}{dx}\right]$$

$$+ \frac{dv_{int}^T}{dx} Y^T v_{int}^a + v_{int}^T \frac{dY^T}{dx} v_{int}^a + v_{int}^T Y^T \frac{dv_{int}^a}{dx} - \frac{dv_{int}^{aT}}{dx} Y v_{int} - v_{int}^{aT} \frac{dY}{dx} v_{int} - v_{int}^{aT} Y \frac{dv_{int}}{dx}$$

$$0 = \frac{di_1}{dx} \cdot 0 + i_1 \cdot 0 + 0 \cdot v_2^a + 0 \cdot \frac{dv_2^a}{dx} - 0 \cdot v_1 - i_1^a \cdot 0 - 0 \cdot v_2 - i_2^a \cdot \underbrace{\frac{dv_2}{dx}}_{= \frac{d\,\text{transfer function}}{dx}}$$

$$+ \frac{dv_{int}^T}{dx} Y^T v_{int}^a + \underbrace{v_{int}^T \frac{dY^T}{dx} v_{int}^a}_{\text{desired}} + v_{int}^T Y^T \cdot 0$$

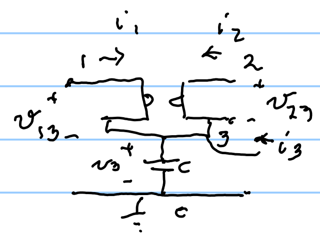
$$- 0 \cdot Y^{aT} v_{int} - v_{int}^{aT} \cdot 0 \cdot v_{int} - \underbrace{v_{int}^{aT} Y \frac{dv_{int}}{dx}}_{= -\left(\frac{dv_{int}^T}{dx} Y^a v_{int}^a\right)}$$

$\Leftarrow$ as the transpose of a scalar is the same as original

cancel if $-Y^a + Y^T = 0$ matrix $\Rightarrow$ choose $Y^a = Y^T$

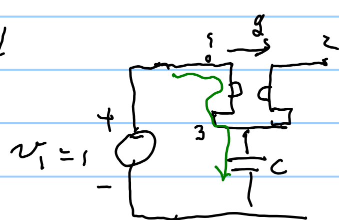
$$0 = v_{int}^{aT} \frac{dY}{dx}^T \cdot v_{int} + i_{2a} \cdot \frac{dv_2}{dx} \Rightarrow \frac{d\, v_2/v_1}{dx} = \frac{-1}{i_{2}^{a}}, v_{int}^T, \frac{dY}{dx}, v_{int}^a$$

choose $i_2^a = -1$ & transpose product



$$Y = \begin{bmatrix} 0 & g & 0 \\ -g & 0 & 0 \\ 0 & 0 & sC \end{bmatrix}, \quad Y^a = Y^T = \begin{bmatrix} 0 & -g & 0 \\ g & 0 & 0 \\ 0 & 0 & sC \end{bmatrix}; \quad \frac{dY}{dC} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & s \end{bmatrix}, \quad \frac{dY}{dg} = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

N



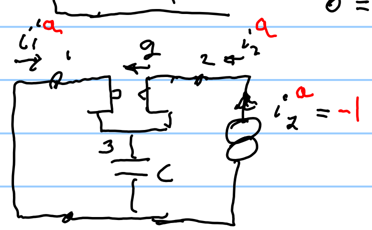
$$\leftarrow i_2 = -g v_{13} = 0 \Rightarrow v_{13} = 0$$

$$i_1 = g v_{23} = i_{cap} = sC v_3 \Rightarrow v_{23} = \frac{sC}{g} v_3 \Rightarrow v_{23} = \frac{sC}{g}$$

$$v_{int} = \begin{bmatrix} 0 \\ sC/g \\ 1 \end{bmatrix}$$

$$0 = -v_1 + v_{13} + v_3 \Rightarrow 0 = -1 + 0 + v_3 \Rightarrow v_3 = 1$$

N^a



$i_2^a = -1$

$$v_{13}^a = -v_3^a \qquad i_1^a = -g v_{23}^a$$

$$i_2^a = g v_{13}^a = -g v_3^a = -1$$

$$\left.\begin{array}{l} i_1^a + i_{2a}^a = i_C^a = sC v_3^a \\ -g v_{23}^a \quad \text{"} -g v_3^a \end{array}\right\} -g v_{23}^a = (g + sC) v_3^a \Rightarrow v_{23}^a = -\frac{g+sC}{g}, v_3^a$$

$$v_{int}^a = \begin{bmatrix} -1/g \\ \frac{-1}{g^2}(g+sC) \\ 1/g \end{bmatrix}$$

$$\frac{d\, v_2/v_1}{dc} = \left[\,0 \quad \frac{\alpha c}{g} \quad 1\,\right] \cdot \underbrace{\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \alpha \end{bmatrix}}_{= Y} \begin{bmatrix} -1/g \\ \frac{-1}{g^2}(g+\alpha c) \\ 1/g \end{bmatrix} = \frac{\alpha}{g} \quad , \quad \frac{d\, v_2/v_1}{dg} = \left[\frac{-1}{g}, \frac{-1}{g^2}(g+\alpha c), \ 1/g\right] \underbrace{\begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}}_{= Y^T} \begin{bmatrix} 0 \\ \alpha c/g \\ 1 \end{bmatrix} = \frac{\alpha c}{g^2}$$