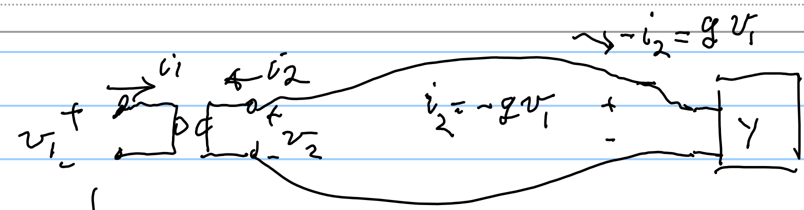
Note Title 11/12/2018

ENEE 610
Tu 11/13/18



$\downarrow$

$i_1 = 0$

open circuit = no load

$i_1 = g v_2 \Rightarrow 0$

$v_1 = \frac{1}{g} i_{\text{into}}$ Y device

$\Downarrow$

output v_o

$\frac{v_o}{v_i} = \frac{N(s)}{D(s)} \Rightarrow D = \frac{N(\infty)}{D(\infty)}$ assume no pole at $s = \infty$

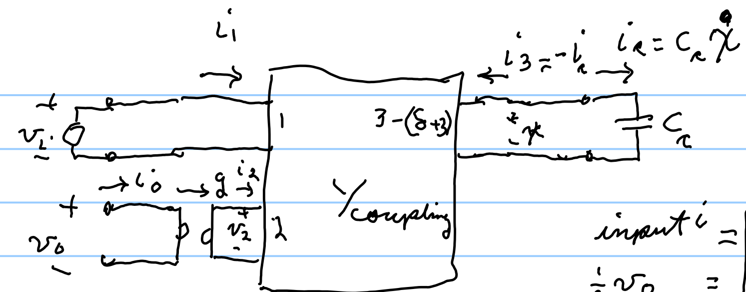
$\Uparrow$

matrix D in state equations = const as v_i & v_o assumed 1×1

single output & input

$$= \frac{b_{\delta-1} s^{\delta-1} + \dots + b_1 s + b_0}{s^\delta + a_{\delta-1} s^{\delta-1} + \dots a_1 s + a_0}$$; a's & b's constant

$\Rightarrow \quad \dot{x} = Ax + Bv_i, \quad v_o = Cx + Dv_i$



$i_2 = \frac{1}{g} v_0$

x = voltage δ-vector

$$\begin{matrix} \text{input } i \\ \frac{1}{g} v_0 \\ -C_x \dot{x} \end{matrix} \begin{matrix} = \\ = \\ = \end{matrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = Y_{coupling} \begin{bmatrix} v_i = \text{input} \\ v_2 = 0 \\ v_3 = x \end{bmatrix} \quad \text{as } i_0 = 0 \text{ for no load}$$

$$\begin{bmatrix} i_i \\ \frac{1}{g} v_0 \\ -C_x \dot{x} \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} & y_{13} \\ \frac{1}{g} D & y_{22} & \frac{1}{g} C \\ -C_x B & y_{32} & -C_x A \end{bmatrix} \begin{bmatrix} v_i \\ 0 \\ x \end{bmatrix}$$

$$\dot{x} = Ax + Bv_i$$

$$v_0 = Cx + Dv_i$$

here $y_{11}, y_{12}, y_{13}, y_{32}$ are don't care matrices

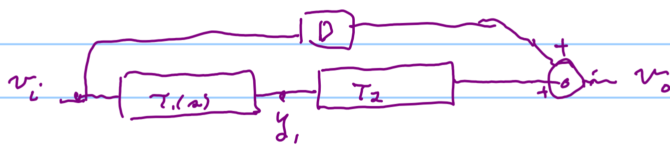
these can be real & arbitrary

$$Y_{coupling} = \left[\begin{array}{cc|c} G_{11} & -\frac{1}{g} D^T & +B^T C_x^T \\ \frac{1}{g} D & G_{22} & \frac{1}{g} C \\ \hline -C_x B & -\frac{1}{g} C^T & -C_x A \end{array}\right]$$

can make a lot of this skew

& passive in 1,1 & 2,2 positions, $G_1 \geq 0$, $G_{22} \geq 0$

Ex: $\frac{v_o}{v_i} = \frac{2s^2+3s+4}{s^3+5s^2+6s+7} + 8$



$T_1 = \frac{1}{s^3+5s^2+6s+7}$, $T_2 = 2s^2+3s+4$

$y_1 = T_1 v_i$, $v_o = T_2 y_1 + D v_i$

For T_1: $(s^3+5s^2+6s+7)\, y_1 = v_i \Rightarrow$ $x_1 = y_1$, $x_2 = \dot{x}_1 = \dot{y}_1 = s y_1$, $x_3 = \dot{x}_2 = \ddot{x}_1 = \ddot{y}_1 = s^2 y_1$

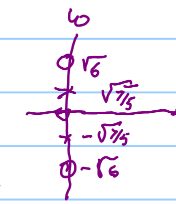
$\underset{\dot{x}_3}{\underset{\shortparallel}{s x_3}} + 5x_3 + 6x_2 + 7x_1 = v_1$ $\Rightarrow \dot{x}_3 = v_i - 7x_1 - 6x_2 - 5x_3$

$$\begin{bmatrix} s & 0 & 0 \\ 0 & s & 0 \\ 0 & 0 & s \end{bmatrix} \overset{\boxed{\cdot}}{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -7 & -6 & -5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} v_i$$

is $P(s) = s^3+5s^2+6s+7$ Hurwitz

$= (5s^2+7)\left[1 + \underbrace{\frac{s^3+6s}{5s^2+7}}_{\text{is LPR?}}\right]$

$y(s) = \frac{s(s^2+6)}{5(s^2+7/5)}$



$\frac{7}{12} = y(1) > 0$, & poles & zeros on $j\omega$ axis & alternate

$$Cx = T_2 y_1 = \underbrace{2A^2 y_1}_{x_3} + \underbrace{3A y_1}_{x_2} + \underbrace{4 y_1}_{x_1} \qquad v_0 = Cx + D v_i$$

$$= \begin{bmatrix} 4 & 3 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$\underset{\text{coupling}}{Y_c} = \begin{bmatrix} G_{11} & -8 & +\begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \\ 8 & G_{22} & \begin{bmatrix} 4 & 3 & 2 \end{bmatrix} \\ -\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} & -\begin{bmatrix} 4 \\ 3 \\ 2 \end{bmatrix} & \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \\ 7 & 6 & 5 \end{bmatrix} \end{bmatrix} \quad 5\times 5$$

choose $C_c = 1_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, $g = 1$

choose $G_{11} = G_{22} = 0$

$$Y_c = \begin{bmatrix} 0 & -8 & 0 & 0 & 1 \\ 8 & 0 & 4 & 3 & 2 \\ 0 & -4 & 0 & -1 & 0 \\ 0 & -3 & 0 & 0 & -1 \\ -1 & -2 & 7 & 6 & 5 \end{bmatrix}$$

5×5 coupling admittance constant, make with OTA's

we can transform the state by $\hat{x} = Px$; $\dot{(Px)} = PAP^{-1}(Px) + PBv_i$ $\Rightarrow \hat{x} = Px$ is a new state

$v_0 = CP^{-1}(Px) + Dv_i$

i.e. load ports 3 through 5 in unit capacitors & port 2 in a unit gyrator, feed port 1 with input voltage good for VLSI