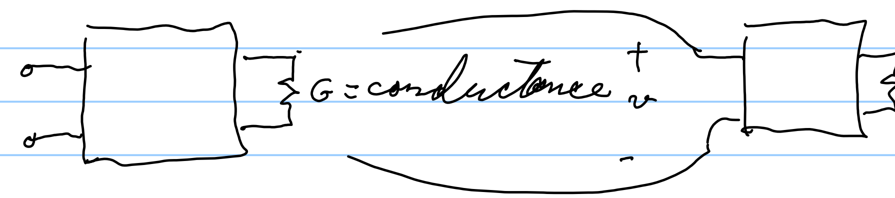
Constant R

$Y_{in} = G \longrightarrow$

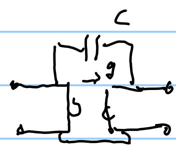


$$Y_{in} = y_{11} - y_{12} \cdot \frac{1}{y_{22}+Y_L} \cdot y_{21} = \frac{y_{11}y_{22} + y_{11}Y_L - y_{12}y_{21}}{y_{22}+Y_L} = \frac{\Delta Y + y_{11}Y_L}{y_{22}+Y_L}$$

for Constant R : $Y_L = G = Y_{in}$ $\qquad G = \frac{\Delta Y + y_{11}G}{y_{22}+G} \Longrightarrow y_{22}G + G^2 = \Delta Y + y_{11}G$

If $y_{11} = y_{22}$ then $y_{11}G = y_{22}G$ $\qquad G^2 = \Delta Y$

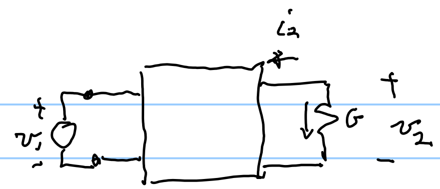
For (Richards' circuit).



$$Y = \begin{bmatrix} sC & -sC+g \\ -sC-g & sC \end{bmatrix}, \text{ as } y_{11} = y_{22} = sC \;\&\; \Delta Y = (sC)^2 - (-sC+g)(-sC-g) = (sC)^2 - ((sC)^2 - g^2) = g^2$$

$\therefore$ as $G^2 = \Delta Y = g^2$ then $\pm g = G \Rightarrow G = |g| > 0$

∴ look @ $-\frac{v_2}{v_1}(s)$ for const. R



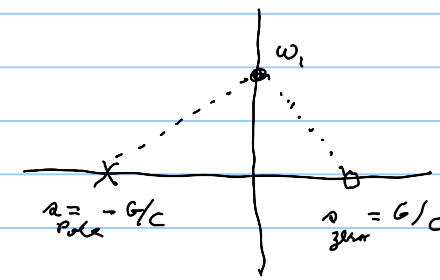
$\Rightarrow -Gv_2 = i_2 = y_{21}v_1 + y_{22}v_2 \Rightarrow (-G - y_{22})v_2 = y_{21}v_1$

$$\frac{v_2}{v_1} = \frac{-y_{21}}{y_{22}+G}$$

for the Richards' circuit $\frac{v_2}{v_1} = \frac{sC+g}{sC+G}$ so if $g=-G$

Then $\frac{v_2}{v_1} = \frac{sC-G}{sC+G} = \frac{s-G/C}{s+G/C}$

∴ can get an all-pass $\frac{v_2}{v_1} = \frac{s-a}{s+a}$, a real $= G/C$

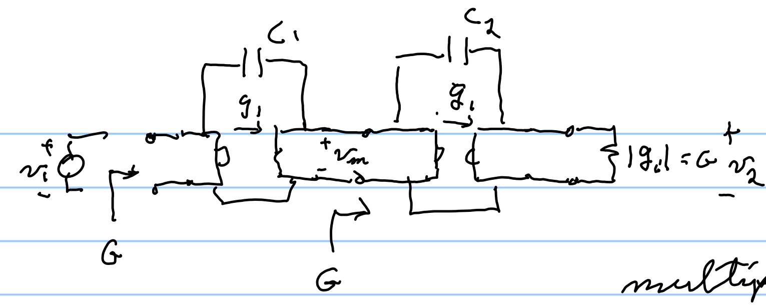


$$\left|\frac{v_2}{v_1}(j\omega)\right| = \left|\frac{j\omega-a}{j\omega+a}\right| = \frac{\sqrt{\omega^2+(-a)^2}}{\sqrt{\omega^2+(a)^2}} = 1$$

∴ use to phase shift

generalizes to $\frac{v_2}{v_1} = \frac{D(-s)}{D(s)} = \frac{Ev\,D - Od\,D}{Ev\,D + Od\,D}$

@ $s=j\omega$, $Ev\,D$ = real part & $Od\,D$ is imaginary $\Rightarrow \left|\frac{D(-j\omega)}{D(j\omega)}\right| = 1$

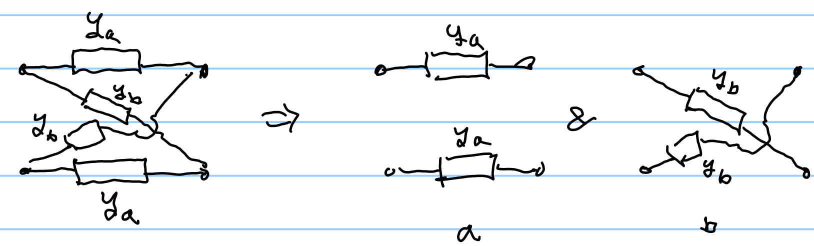


$$\frac{v_m}{v_1} = \frac{s - C_1/G}{s + C_1/G} \qquad \frac{v_2}{v_m} = \frac{s - C_2/G}{s + C_2/G}, \quad \delta\left[\frac{v_2}{v_1}\right] = 2 \text{ uses } 2C's$$

multiply $\quad \frac{v_2}{v_m} \times \frac{v_m}{v_1} = \frac{v_2}{v_1} = \left(\frac{s - C_2/G}{s + C_2/G}\right)\left(\frac{s - C_1/G}{s + C_1/G}\right)$

$\therefore$ gives higher order but only with real zeroes and poles

another way to get all-pass (the classical way), the constant R lattice



$$\Rightarrow Y_a + Y_b \Rightarrow Y_a = \begin{bmatrix} y_a/2 & -y_a/2 \\ -y_a/2 & y_a/2 \end{bmatrix}, \quad Y_b = \begin{bmatrix} y_b/2 & +y_b/2 \\ y_b/2 & y_b/2 \end{bmatrix}$$

$$Y = Y_a + Y_b = \frac{1}{2}\begin{bmatrix} y_a + y_b & y_b - y_a \\ y_b - y_a & y_a + y_b \end{bmatrix}, \quad 4\underbrace{\Delta Y}_{\text{determinant}} = (y_a + y_b)^2 - (y_b - y_a)^2 = 2y_a y_b - (-2y_a y_b) = 4y_a y_b \Rightarrow \Delta Y = y_a y_b = G^2$$

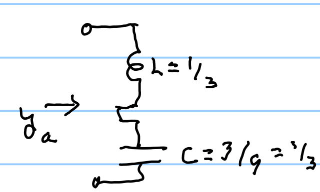
$\therefore$ for constant R; $y_a y_b = G^2 \Rightarrow y_b = G^2/y_a$

$$\frac{v_2}{v_1}(s) = \frac{-y_{21}}{y_{22}+G} = \frac{-(y_b-y_a)/2}{[(y_a+y_b)/2]+G} = \frac{-(G^2/y_a - y_a)}{2G+(y_a+G^2/y_a)} = \frac{y_a^2 - G^2}{y_a\left(\frac{2Gy_a+y_a^2+G^2}{y_a}\right)} = \frac{y_a^2-G^2}{(y_a+G)^2} = \frac{y_a-G}{y_a+G}$$

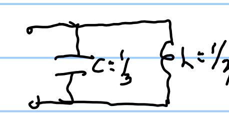
- If $y_a = LPR = \frac{n_a}{d_a} \Rightarrow \frac{v_2}{v_1} = \frac{n_a - G d_a}{n_a + G d_a} \Rightarrow$ all-pass of degree of $y_a \Rightarrow \delta[y_a] = \delta\left[\frac{v_2}{v_1}\right]$; $\delta[\cdot]$ = degree of

Ex: $\frac{s^2-3s+9}{s^2+3s+9} = \frac{v_2}{v_1} \Rightarrow \frac{v_2}{v_1} = \frac{(s^2+9)(1-3s/(s^2+9))}{(s^2+9)\left(1+\frac{3s}{s^2+9}\right)} = \frac{-\left(\frac{3s}{s^2+9}-1\right)}{\left(\frac{3s}{s^2+9}+1\right)}$; $G=1$

$y_a = \frac{3s}{s^2+9} = \frac{1}{\frac{s}{3}+\frac{9}{3s}} \Rightarrow$



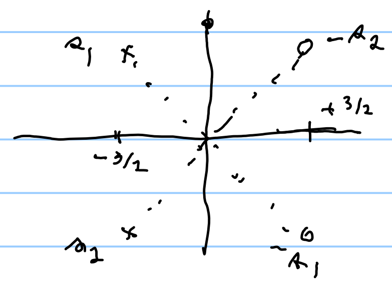
$y_b = \frac{G^2}{y_a} = \frac{1}{\frac{3s}{s^2+9}} = \frac{s^2+9}{3s} = \frac{s}{3}+\frac{3}{s}$



$$v_2 = \frac{s^2 - 3s + 9}{s^2 + 3s + 9} \cdot v_1 \qquad 1\delta\left[\frac{v_2}{v_1}\right] = 2, \quad \delta[y_s] = 2 \qquad y_s = \frac{3s}{s^2+9}$$

uses 8 L's & C's, 4 L's & 4 C's

poles @ $s_{1,2} = -\frac{3}{2} \pm \frac{1}{2}\sqrt{9-36} = -\frac{3}{2} \pm \frac{1}{2}j\sqrt{27}$



is all-pass