ENEE 610
Tu 10/30/18

Note Title 10/28/2018

$$Y(s) = \frac{5s^2+4s+8}{2s^2+s+2}$$ if PR desire to synthesize by Bott-Duffin scheme

to find max. & min on $j\omega$ axis of $\mathrm{Re}\,Y(s)$, $\sigma \geq 0$

take $e^{Y(s)}$ & $e^{-Y(s)}$ $e^{Y(s)} = e^{\mathrm{Re}Y + j\mathrm{Im}Y} = e^{\mathrm{Re}Y} \cdot e^{j\mathrm{Im}Y} \Rightarrow |e^{Y(s)}| = e^{\mathrm{Re}Y} \cdot 1$

$|e^{-Y(s)}| = e^{-\mathrm{Re}Y} \cdot 1 \Rightarrow$ min of $\mathrm{Re}\,Y(j\omega)$ occurs on $j\omega$ axis if $Y(s)$ is analytic in $\sigma \geq 0$

$\Rightarrow$ use max modulus theorem

Find $\mathrm{Re}\,Y(j\omega)$ is minimum, @ $\omega_0^2 = 2$; then $Y(j\omega_0) = 2 + jB(\omega_0)$ $\omega_0 = +\sqrt{2}$

freq. admittance

$Y_m(s) = Y(s) - 2$, $\mathrm{Re}\,Y_m(j\omega_0) = 0$, & $\mathrm{Re}\,Y(j\omega) \geq 0$ so Y_m is PR

$$y_m(s) = \frac{s^2+2s+4}{2s^2+s+2} \; ; \quad y_m(j\omega_0) = 0 - 1.414 \cdot j \qquad j = \sqrt{-1}$$

Force this to zero by using the Richards' function

$$y_n(s) = y_m(k)\left[\frac{k\,y_m(k) - s\,y_m(s)}{k\,y_m(s) - s\,y_m(k)}\right]$$

$y_n(j\omega_0) \Rightarrow$ numerator is

$k\,y_m(k) - j\omega_0 \cdot (-1.414)j = k\,y_m(k) - \omega_0(1.414)$

$= k\,y_m(k) - 2$ desire a $k > 0$ to make this 0

$$\frac{y_n}{y_m(k)} =$$

$\therefore$ find k for $k\,y_m(k) = 2 \Rightarrow k = k_z = 0 \Rightarrow y_n(s)\big| = 0$; $y_n(s)$ is rational, not $\equiv 0$

$s = j\omega_0$ & $s = -j\omega_0$ as rational with real coefficients

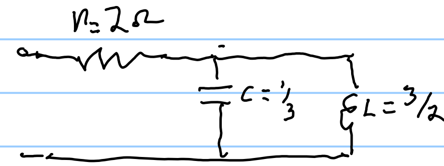
$\therefore$ $y_n(s) \Rightarrow (s^2+\omega_0^2)$ factors as we forced $y_n(j\omega_0) = 0$ by $k = k_z$

the numerator

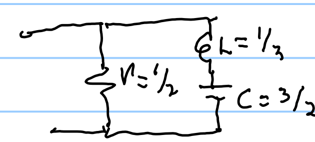
also $s = k_z$ factors $\quad y_n(s) = \dfrac{k(s^2+\omega_0^2)(s-\overset{k_z}{2})}{(s-2)(as^2+bs+c)}$, $y_m(k_z) = \dfrac{\overset{= k_z^2}{4} + 2 \cdot 2 + 4}{2 \times 4 + 2 + 2} = \dfrac{12}{12} = 1$

$$y_n(s) = 1\cdot\left(\frac{(s-2)(s^2+2)}{(s-2)(2s^2+3s+4)}\right) = \frac{s^2+2}{(2s^2+4)+3s} \Rightarrow \frac{1}{y_n(s)} = 2 + \frac{3s}{s^2+2} = 2 + \cfrac{1}{\frac{1}{3}s + \cfrac{1}{\frac{3}{2}s}} \Leftarrow s\,z$$

If $y_n(s)$ is an admittance then $y_n \Rightarrow$



If y_n is an impedance $z_n = \dfrac{s^2+2}{(2s^2+4)+3s}$ then



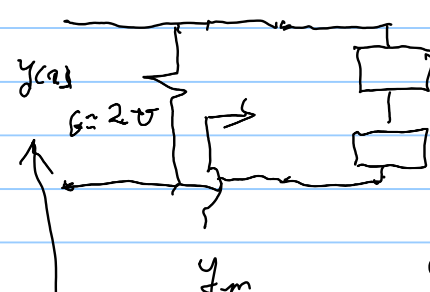
$$\Rightarrow \frac{y_n(s)}{y_m(s)} = \frac{1 - \frac{s}{k}\cdot\frac{y_m(s)}{y_m(k)}}{\frac{y_m(s)}{y_m(s)} - \frac{s}{k}} \Rightarrow \hat{y}_n(s) = y_n(s)/y_m(k), \quad \hat{y}_m(s) = y_m(s)/y_m(k)$$

$$\hat{y}_n = \frac{1 - \frac{s}{k}\hat{y}_m}{\hat{y}_m - s/k} \Rightarrow \hat{y}_n\left(\hat{y}_m - \frac{s}{k}\right) = 1 - \frac{s}{k}\hat{y}_m$$

$$\hat{y}_m\left(\hat{y}_n + \frac{s}{k}\right) = 1 + \frac{s}{k}\hat{y}_n \Rightarrow \hat{y}_m(s) = \frac{1 + \frac{s}{k}\hat{y}_n}{(s/k) + \hat{y}_n} = \cfrac{1}{\cfrac{s/k}{1 + s/k\,\hat{y}_n} + \cfrac{\hat{y}_n}{1 + s/k\cdot\hat{y}_n}}$$

end circuit

$$\frac{y_m(s)}{y_m(k)} = \frac{1}{\dfrac{1}{\frac{k}{s}+\hat{y}_n} + \dfrac{1}{\frac{1}{\hat{y}_n}+\frac{s}{k}}} \Rightarrow$$

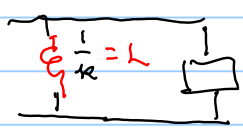


$$Z_1 = \frac{1}{\frac{k}{s}+\hat{y}_n} \leftarrow y_i$$

$$Z_2 = \frac{1}{\frac{1}{\hat{y}_n}+\frac{s}{k}}$$

$a\,Z$

$$Z_1 = \frac{1}{\frac{k}{s}+\hat{y}_n}$$



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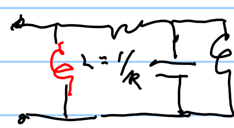
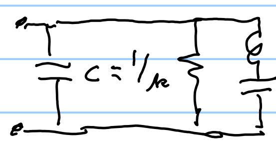


$$Z_2 = \frac{1}{\frac{1}{\hat{y}_n}+\frac{s}{k}} \Rightarrow$$



$$y(s) = \frac{5s^2+4s+8}{2s^2+s+2}$$

use 6 reactive elements