Note Title

ENEE 610
Th 10/25/18

10/24/2018

$$y_I(\alpha) = y_i(k)\left[\frac{k\,y_i(k) - \alpha\,y_i(\alpha)}{k\,y_i(\alpha) - \alpha\,y_i(k)}\right]$$

$y(\alpha) = \frac{3\alpha}{\alpha^2+4}$ choose any real $k>0$, $k=3$, $y(k) = \frac{9}{9+4} = \frac{9}{13}$

$g = y(k)$

$C = \frac{g}{k} = \frac{y(k)}{k}$ $\quad C = \frac{9/13}{3} = 3/13$

$$y_L = y(k)\left[\frac{k\,y(k) - \alpha\,y(\alpha)}{k\,y(\alpha) - \alpha\,y(k)}\right] = \frac{9}{13}\left[\frac{3\times\frac{9}{13} - \frac{3\alpha^2}{\alpha^2+4}}{\frac{9\alpha}{\alpha^2+4} - \alpha,\frac{9}{13}}\right] = \frac{9}{13}\left[\frac{\frac{27}{13}\alpha^2 + \frac{4\times 27}{13} - 3\alpha^2}{9\alpha - \left(\frac{9}{13}\alpha^3 + \frac{4\times 9}{13}\alpha\right)}\right]$$

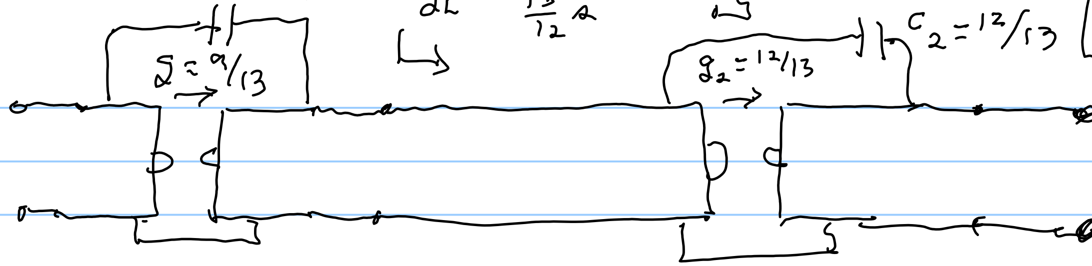
$$\begin{array}{r} 39 \\ -27 \\ \hline \end{array} \qquad \begin{array}{r} 4\times 27 \\ \hline 108 \end{array}$$

$$= \frac{9}{13}\left[\frac{\frac{27-3\times 13}{13}\alpha^2 + \frac{4\times 27}{13}}{-\frac{9}{13}\alpha^3 + \left(\frac{9\times 13}{13} - \frac{4\times 9}{13}\right)\alpha}\right] = \frac{9}{13}\left[\frac{-12\alpha^2 + 12\times 9}{\alpha\left(-9\alpha^2 + 9\times 9\right)}\right] = \frac{9}{13}\times\frac{12}{9}\left(\frac{-\alpha^2+9}{-\alpha^2+9}\right)\frac{1}{\alpha}$$ shows α^2-9 divides

$$= \frac{12}{13}\times\frac{1}{\alpha}$$ now repeat

$y = \dfrac{3a}{a^2+4}$

$y_L = \dfrac{1}{\frac{13}{12}a} = \;$ $L = 13/12$ $\quad y_{LL} = 0$



$y_L = \dfrac{1}{\frac{13}{12}a} =$

$C_2/g_2^2 = \dfrac{1}{g_2 k_2}$

repeat on $y_2 \Rightarrow \quad y_{LL} = y_L(k_2)\left[\dfrac{k_2 y_L(k_2) - a\, y_L(a)}{k_2 y_L(a) - a\, y_L(k_2)}\right]$

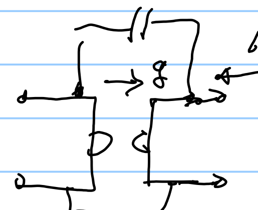
choose $k_2 = 1$

$y_L(k_2) = 12/13 = g_2$

$C \Rightarrow g_2/k_2 = 12/13$

$y_{LL} = \dfrac{12}{13}\left[\dfrac{1\cdot 12/13 - a\cdot\frac{12}{13a}}{1\cdot\frac{12}{13a} - a\cdot 12/13}\right] \equiv 0$

look at the 2-port



$i_2 = 0$ for an open circuit load

$v = Zi$

$Z_{2\text{-port}} = Y^{-1}_{2\text{-port}} = \begin{bmatrix} aC & -aC+g \\ -aC-g & aC \end{bmatrix}^{-1} \; ; \quad Z = \dfrac{1}{g^2}\begin{bmatrix} aC & aC-g \\ aC+g & aC \end{bmatrix}$

g^2 = determinant

$v_1 =$

$$v = Zi = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} sC/g^2 & \frac{sC-g}{g^2} \\ \frac{sC+g}{g^2} & sC/g^2 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 = 0 \end{bmatrix} \Rightarrow v_1 = \frac{sC}{g^2} \cdot i_1 \Rightarrow \text{input } z = \frac{1}{\text{input} = y_L}$$

$\Downarrow$

inductor

$L = C/g^2$

given $y(s)$ as LPR then $y_R(s)$ is LPR

$2\,\mathrm{Ev}\, y = y(s) + y(-s) = 0$ $\qquad 2\,\mathrm{Ev}\, y_R$

$$\mathrm{Ev}\, y_R: \quad \frac{k y(k) - s y(s)}{k y(s) - s y(k)} = y_R(s) \quad , \quad y_R(-s) = \frac{k y(k) - (-s) y(-s)}{k y(-s) - (-s) y(k)} = \frac{k y(k) + s y(-s)}{k y(-s) + s y(k)}$$

$$= \frac{k y(k) - y(s)}{-k y(-s) + s y(k)} = -y_R(s)$$

$\Rightarrow \mathrm{Ev}\, y_R(s) \equiv 0$ if $y(s)$ is LPR

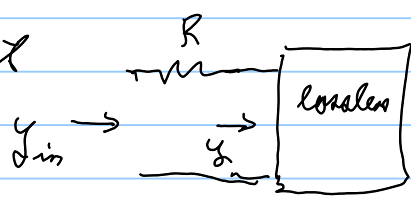
Bott-Duffin synthesis p. 361

$$\frac{y_R}{y(k)} = \frac{k\,y(k) - s\,y(s)}{k\,y(s) - s\,y(k)} = \frac{1 - \frac{s}{k}\cdot y/y(k)}{y/y(k) - s/k} \Rightarrow \left(\frac{y}{y(k)} - \frac{s}{k}\right)\frac{y_R}{y(k)} = 1 - \frac{s}{k}\,y/y(k)$$

$$\left(\frac{y_R}{y(k)} + \frac{s}{k}\right)\frac{y}{y(k)} = 1 + s/k\cdot\frac{y_R}{y(k)} \Rightarrow \frac{y}{y(k)} = \frac{1 + \frac{s}{k}\cdot y_R/y(k)}{\frac{s}{k} + y_R/y(k)}$$

$$\frac{y}{y(k)} = \frac{1}{s/k + y_R/y(k)} + \frac{\frac{s}{k}\cdot y_R/y(k)}{\frac{s}{k} + y_R/y(k)} = \frac{1}{\frac{s}{k} + \frac{y_R}{y(k)}} + \frac{1}{\frac{1}{y_R/y_k} + \frac{1}{s/k}}$$

Return to Hurwitz test



$$y_{in} = \frac{1}{R + 1/y}\Bigg|_{R=1,\ y=n/d} = \frac{d(s)}{n(s)+d(s)} = \frac{d(s)}{P(s)}$$

$P(s) = n(s) + d(s) = Ev\,P + Od\,P$ $\qquad$ $Od\,P(s) = -Od\,P(-s) = s\,P_2(s^2)$

$= (s^2+a^2)\,n_2(s) + (s^2+a^2)\,d_2(s)$

Ex: $\quad P(s) = (s^2+3)(s^2+4) + s\underbrace{(s^2+3)(s^2+5)}_{P_2(s^2)}$; here $Ev\,P$ & $Od\,P$ have s^2+3 common

form $y(s) = \dfrac{Ev\,P}{Od\,P} = \dfrac{(s^2+3)(s^2+4)}{s(s^2+3)(s^2+5)} = \underbrace{\dfrac{s^2+4}{s(s^2+5)}}$ $\Leftarrow$ common even factor cancels

is LPR as zeros & poles alternate on $j\omega$ axis & $y(1) = 5/6 > 0$

here $\delta[P(s)] = 5$

$\delta[y(s)] = 3$ as $3 < 5$ we know there is a common even factor

form $y(s)+1$ & get its $n_2 + d_2 = (s^2+4) + s(s^2+5) = s^3 + s^2 + 5s + 4$

$$P(s) = (s^4 + 7s^2 + 12) + (s^5 + 8s^3 + 15s) = s^5 + s^4 + 8s^3 + 7s^3 + 15s + 12$$

$$= (s^2+3)(s^3+s^2+5s+4)$$

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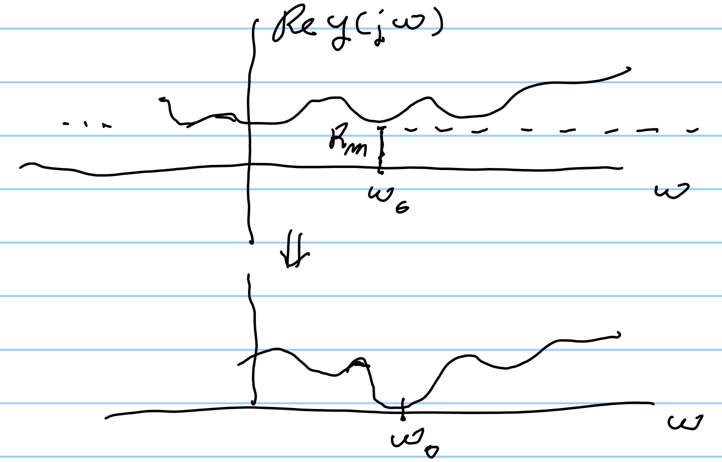
Bott-Duffin $\Rightarrow$ 1st create a "minimum" $y(s)$ (PR)

$\Rightarrow$ $\mathrm{Re}\, y(j\omega_0) = 0$ for some ω_0



find minimum

$$y(s) - R_m = y_{\text{minimum}}(s)$$

@ a real ω_0, $0 < \omega_0 < \infty$

$$y_M = y_{\text{minimum}}(j\omega_0) = jB \quad \text{purely imaginary}$$

look @ $Y_R(j\omega_0) = \dfrac{k\,Y_M(k) - j\omega_0 Y_M(j\omega_0)}{k\,Y_M(j\omega_0) - j\omega_0 \cdot Y_M(k)}$ $\Longrightarrow$ numerator $k\,Y_M(k) + \omega_0 B$

denominator $jkB - j\omega_0 Y_M(k)$

always a real positive k forcing $Y_R(j\omega_0) = 0$ if $B < 0$

$\dfrac{1}{Y_R(j\omega_0)} = 0$ if $B > 0$