Note Title 10/23/2018

ENEE 610
10/23/18

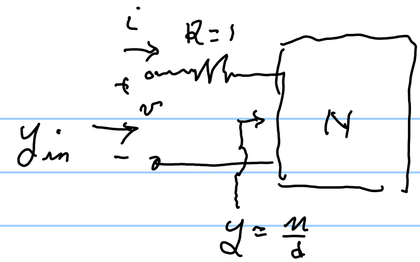
Richards' function $\quad y_L(s) = y_i(k)\left[\dfrac{k\,y_i(k) - s\,y_i(s)}{k\,y_i(s) - s\,y_i(k)}\right]$

polynomials: $\quad P(s) = \sum_{i=0}^{\delta} a_i \cdot s^i \qquad \delta = \text{degree}$

Hurwitz: no zeroes in $\sigma > 0$, $\quad s = \sigma + j\omega$ } stable system
& only simple zeroes on $\sigma = 0$ ($j\omega$ axis) ← if none on $j\omega$ axis call strictly Hurwitz

$Ev\, P(s) = \frac{1}{2}\left(P(s) + P(-s)\right) \qquad$ (removes s^{2k+1} terms

$Od\, P(s) = \frac{1}{2}\left(P(s) - P(-s)\right) \qquad \Rightarrow Ev\,P(s) + Od\,P(s) = P(s)$



$$y_{in} = \frac{1}{z_{in}} = \frac{1}{1+y} = \frac{1}{1+\frac{n(s)}{d(s)}} = \frac{d(s)}{n(s)+d(s)}$$

$$i = y_{in} v \Rightarrow i = \frac{d(s)}{n(s)+d(s)} \cdot v = \frac{d(s)}{P(s)} \cdot v$$ if $v = 0$ (input = short) $\Rightarrow$ $P(s)=0$ can give nonzero $i(s)$

$\Rightarrow$ zeroes of $P(s) = n(s) + d(s)$ are short circuit natural frequencies

if N is lossless then $y = \text{LPR} = \frac{n}{d}$ is even/odd or odd/even

$\Rightarrow$ R requires $P(s) = n + d$ to be Hurwitz $\quad P(s) = \text{Ev}\,P(s) + \text{Od}\,P(s) = \text{Ev}\,P(s)\left[1 + \frac{\text{Od}}{\text{Ev}} P(s)\right]$

$\therefore$ given a polynomial $P(s)$ form $y(s) = \frac{\text{Od}\,P}{\text{Ev}\,P}$ & if this is LPR then

$P(s)$ is Hurwitz except for common even polynomials which can factor out of $y(s)$

$$P(s) = s^4 + 2s^3 + 3s^2 + 2s + 1 = (s^4+3s^2+1) + (2s^3+2s) = (s^4+3s^2+1) + s(2s^2+2)$$
$$= (s^4+3s^2+1)\left[1 + \frac{2s^3+2s}{s^4+3s^2+1}\right]$$

is $y(s) = \dfrac{2s^3+2s}{s^4+3s^2+1}$ an LPR function $\Rightarrow$ try 1st Cauer

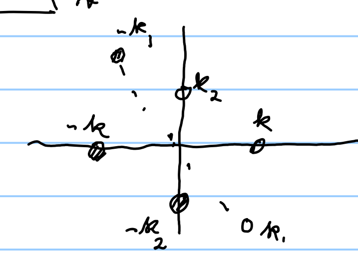
$y(s) = \dfrac{1}{z(s)}$

$$\begin{array}{l} 2s^3+2s \,\big)\, s^4+3s^2+1 \;\; \left(\tfrac{1}{2}s\right. \\ \qquad\quad\;\; s^4+s^2 \\ \qquad\quad\;\; 2s^2+1 \,\big)\, 2s^3+2s \;\; \left(s\right. \\ \qquad\qquad\qquad\quad\;\; 2s^3+s \\ \qquad\qquad\qquad\quad\;\; s \,\big)\, 2s^2+1 \;\; \left(2s\right. \\ \qquad\qquad\qquad\qquad\;\; 2s^2 \\ \qquad\qquad\qquad\qquad\;\; 1 \,\big)\, s \;\; \left(s\right. \end{array}$$

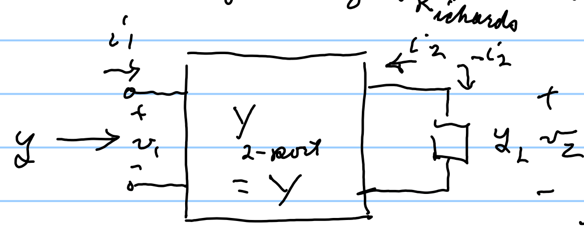
$\Big\}$ $y(s)$ is LPR $\Rightarrow P(s)$ is Hurwitz
$\wedge$
strictly

For Richards functions:

$s = \pm k$ cancels if k is a zero of $Ev\, y(s)$; $\Rightarrow y(s) + y(-s) = 0$



for LPR, $Ev\, y(s) \equiv 0$ $\therefore$ any real positive k is a zero of $Ev\, y(s)$
this allows the degree of $y_{R_{ichards}}$, $\delta[y_R] = \delta[y] - 1$, to go down



$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \Rightarrow \begin{bmatrix} \\ -i_2 = y_L v_2 \end{bmatrix}$$

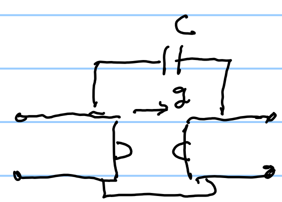
$\Rightarrow -y_L v_2 = y_{21} v_1 + y_{22} v_2 \Rightarrow -(y_L + y_{22}) v_2 = y_{21} v_1$

$$v_2 = -\frac{y_{21}}{y_L + y_{22}} \cdot v_1 \Rightarrow i_1 = y_{11} v_1 + y_{12}\left(\frac{-y_{21}}{y_L + y_{22}}\right) v_1 = \frac{[y_{11} y_L + y_{11} y_{22} - y_{12} y_{21}]}{y_L + y_{22}} v_1$$

$$y = \frac{y_{11} y_L + \Delta_Y}{y_L + y_{22}}, \quad \Delta_Y = \text{determinant } Y = y_{11} y_{22} - y_{12} y_{21}$$

$$\Rightarrow y \cdot y_L + y \cdot y_{22} = y_{11} \cdot y_L + \Delta_Y \Rightarrow (y - y_{11}) y_L = \Delta_Y - y y_{22} \Rightarrow y_L = \frac{\Delta_Y - y_{22} \cdot y}{y - y_{11}}$$

circuit to give Richards' function for y_L when load



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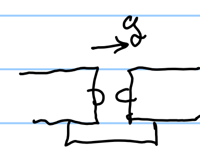

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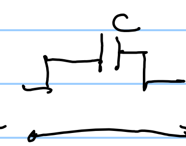

$$Y = \begin{bmatrix} sC & g - sC \\ -g - sC & sC \end{bmatrix}; \quad \Delta_Y = (sC)^2 - (g - sC)(-g - sC) = (sC)^2 + (g^2 - (sC)^2) = g^2$$

$$Y_g = \begin{bmatrix} 0 & g \\ -g & 0 \end{bmatrix} \qquad Y_c = \begin{bmatrix} sC & -sC \\ -sC & sC \end{bmatrix}$$

$$\therefore \quad y_L = \frac{\Delta y - y_{22} y}{y - y_{11}} = \frac{g^2 - \lambda C\, y(\lambda)}{y(\lambda) - \lambda C} = \frac{g^2\left(1 - \frac{\lambda C}{g}\cdot\frac{y(\lambda)}{g}\right)}{g\left(\frac{y(\lambda)}{g} - \frac{\lambda C}{g}\right)} = g\left(\frac{1 - \left(\frac{\lambda C}{g}\right)\left(\frac{y(\lambda)}{g}\right)}{\left(\frac{y(\lambda)}{g}\right) - \frac{\lambda C}{g}}\right)$$

$$y_L(\lambda) = y_i(k)\left[\frac{k\, y_i(k) - \lambda\, y_i(\lambda)}{k\, y_i(\lambda) - \lambda\, y_i(k)}\right] \Rightarrow \frac{y_L(\lambda)}{y(k)} = \frac{k\, y(k)\left[1 - \frac{\lambda}{k}\cdot\frac{y(\lambda)}{y(k)}\right]}{k\, y(k)\left[\frac{y(\lambda)}{y(k)} - \frac{\lambda}{k}\right]} = \left(\frac{1 - \frac{\lambda}{k}\cdot\frac{y(\lambda)}{y(k)}}{\left(\frac{y(\lambda)}{y(k)}\right) - \frac{\lambda}{k}}\right)$$

compare

$$\Rightarrow \frac{g}{C} = k, \quad g = y(k)$$

$$C = \frac{g}{k} = \frac{y(k)}{k}$$

$\therefore y(\lambda)$ results by loading the gyrator-C 2-port in $y_L(\lambda)$

Example: $y(\lambda) = \frac{3\lambda}{\lambda^2+4}$ choose any real $k > 0$, $k = 3$, $y(k) = \frac{9}{9+4} = \frac{9}{13}$

$$y_L = y(k)\left[\frac{k\, y(k) - \lambda\, y(\lambda)}{k\, y(\lambda) - \lambda\, y(k)}\right] = \frac{9}{13}\left[\frac{3\times\frac{9}{13} - \frac{3\lambda^2}{\lambda^2+4}}{\frac{9\lambda}{\lambda^2+4} - \lambda\cdot\frac{9}{13}}\right] = \frac{9}{13}\left[\frac{\frac{27}{13}\lambda^2 + \frac{4\times 27}{13} - 3\lambda^2}{9\lambda - \left(\frac{9}{13}\lambda^3 + \frac{4\times 9}{13}\lambda\right)}\right]$$

and $x^2 - k^2$ cancels, as k is a zero of $\text{Ev}\, y(x) \Rightarrow x^2 - 9$ cancels