ENEE 610
10/18/18

Note Title 10/18/2018

$$y(s) = \frac{s(s^2+3)}{(s^2+1)(s^2+4)} = \frac{s^3+3s}{(s^4+5s^2+4)} = \frac{\text{Odd}}{\text{Even}}$$

$\Rightarrow$ any LPR function is Ev/odd or $\frac{\text{odd}}{\text{Ev}}$

& a pole or a zero @ $\infty = s$

& a pole or a zero @ $s = 0$

1st Cauer $\Rightarrow$ remove poles @ $s = \infty$

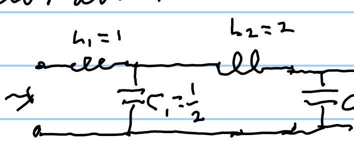
2nd " " $\Rightarrow$ " " @ $s = 0$

Ex: $y(\infty) = 0 \Rightarrow$ so turn over $z(s) = \frac{s^4+5s^2+4}{s^3+3s} = \frac{s^4}{s^3} + \text{remainder} = s + \underbrace{\left(\frac{s^4+5s^2+4}{s^3+3s} - s\right)}_{\frac{2s^2+4}{s^3+3s}}$

$$\begin{array}{r|l} s^3+3s & s^4+5s^2+4 \quad (s) \\ & s^4+3s^2 \\ \hline 2s^2+4 & s^3+3s \quad (\tfrac{1}{2}s) \\ & s^3+2s \\ \hline 1s & 2s^2+4 \quad (2s) \\ & 2s^2+4 \;/\; \frac{4}{2s} = \frac{1}{\frac{1}{2}s} \end{array}$$

$$= s + \cfrac{1}{\frac{1}{2}s + \cfrac{1}{2s + \cfrac{1}{\frac{1}{2}s}}}$$ continued fraction expansion about ∞

1st Cauer

$y, z \rightarrow$ 

$\Rightarrow$ use for low pass filters

for 2nd Cauer divide lowest power of s into lowest power

$$y(s) = \frac{3s + s^3}{4 + 5s^2 + s^4} \Rightarrow z = \frac{1}{y} = \frac{4 + 5s^2 + s^4}{3s + s^3}$$

$3s + s^3 \,\big)\, 4 + 5s^2 + s^4$ quotient: $4/(3s)$

$4 + \frac{4s^2}{3}$

$\frac{11}{3}s^2 + s^4 \,\big)\, 3s + s^3$ quotient: $\frac{9}{11s}$

$3s + \frac{9}{11}s^3$

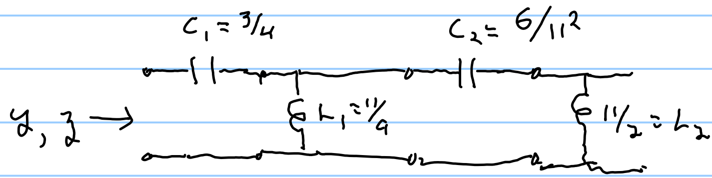
$\frac{2}{11}s^3 \,\big)\, \frac{11}{3}s^2 + s^4$ quotient: $\frac{11^2}{6s}$

$\frac{11}{3}s^2$

$s^4 \,\big)\, \frac{2}{11}s^3$ quotient: $\frac{2}{11s}$

continued fraction about $s = 0$

$$y(s) = 0 + \cfrac{1}{\frac{4}{3s} + \cfrac{1}{\frac{9}{11s} + \cfrac{1}{\frac{11^2}{6s} + \cfrac{1}{\frac{2}{11s}}}}}$$



2nd Cauer

zeros of transmission @ $s = 0$ $\Rightarrow$ used for high pass filters

Richards' function $\quad y_L(s) = y_i(k)\left[\dfrac{k y_i(k) - s\, y_i(s)}{k\, y_i(s) - s\, y_i(k)}\right]$ has a cancellation @ $s = k$

(P.361 of text)

$y_L(s)$ = Richards' $y(s)$ is PR if $y_i(s)$ is PR & k is a real positive constant

$k\, y_i(s)$ is also PR. $\Rightarrow y_i(k) > 0$ ($y_i(k)$ is real & no pole or zero of $y(s)$ is in $\sigma > 0$)

$\therefore$ normalize by $y_i(k) \Rightarrow y_L(s)/y_i(k) = y_R$; $\dfrac{y_i(s)}{y_i(k)} = y$

$$y_R = \frac{k - s y}{k y - s} \Rightarrow \text{convert to scattering} \Rightarrow \text{BR for } S(s) \text{ if } y(s) \text{ is PR}$$

$$S_R = \left(\frac{1 - y_R}{1 + y_R}\right) = \left(\frac{(ky - s) - (k - sy)}{(ky - s) + (k - sy)}\right) = \frac{(k)(y-1) + s(-1+y)}{(k)(y+1) + s(-1-y)} = \frac{(y-1)(k+s)}{(y+1)(k-s)}$$

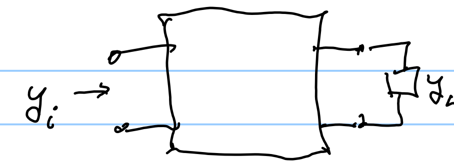
$$= (-1)\underbrace{\left(\frac{1-y}{1+y}\right)}_{BR}\underbrace{\left(\frac{k+s}{k-s}\right)}_{BR}.$$

here $1 - y(s) = 1 - \dfrac{y_i(s)}{y_i(k)} = 0$ @ $s = k$

$\therefore$ $k - s$ cancels

look on $j\omega$ axis $\quad |S_R(j\omega)| = |-1| \cdot \left|\dfrac{1 - y(j\omega)}{1 + y(j\omega)}\right| \cdot \left|\dfrac{k + j\omega}{k - j\omega}\right| = 1\,(\leq 1)\left(\dfrac{\sqrt{k^2+\omega^2}}{\sqrt{k^2+\omega^2}}\right) \leq 1$

$\therefore$ $S_R(s)$ is BR $\Rightarrow$ y_R is PR to synthesize y_i we load a 2-port in y_L



look for k to give cancellation
of $s = -k$ (along with that of $s = k$) $\Longrightarrow$ $\delta(y_L) = \delta(y_i) - 1$, δ = degree = highest power of s in y_i

numerator: $k\,y_i(k) - s\,y_i(s) = 0$ @ $s = -k \Longrightarrow k\,y_i(k) - (-k)\,y(-k) = 0 \Rightarrow k\left(y_i(k) + y_i(-k)\right) = 0$
$\Longrightarrow y_i(k) = -y_i(-k) \iff k$ is a zero of $Ev\, y_i(s)$

denominator: $k\,y_i(s) - s\,y_i(k) = 0$ @ $s = -k \Rightarrow k\,y_i(-k) - (-k)\,y_i(k) = 0 \Rightarrow k\left(y_i(-k) + y_i(k)\right) = 0$

$\therefore$ can get a degree reduction by 1 if k is a zero of $Ev(y_i(s)) = \dfrac{y(s) + y(-s)}{2}$ with $k > 0$

if $y_i(s)$ is LPR any $k > 0$ works.