Note Title 10/15/2018

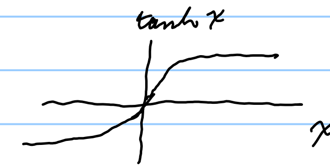
ENEE 610
10/16/18

$j = \sqrt{-1}$ $\quad \cos x = \dfrac{e^{jx} + e^{-jx}}{2} \Rightarrow \cos(jx) = \dfrac{e^{-x} + e^{+x}}{2} = \cosh x$

$\sin x = \dfrac{e^{jx} - e^{-jx}}{2j} \Rightarrow -j\sin(jx) = -j \cdot \dfrac{e^{-x} - e^{+x}}{2j} = \dfrac{e^{x} - e^{-x}}{2} = \sinh x$

$\tan x = \sin x / \cos x$

$\tanh x = \dfrac{\sinh x}{\cosh x}$; $\dfrac{d \tanh x}{dx} = 1 - \tanh^2 x$



meromorphic $\Rightarrow$ singularities are poles (& zeroes)

positive real $y(s)$: y_1 & y_2 positive real then $1/y_1$ is positive real

$y_1 + y_2$ is positive real

$y_1(y_2(s))$ is positive real

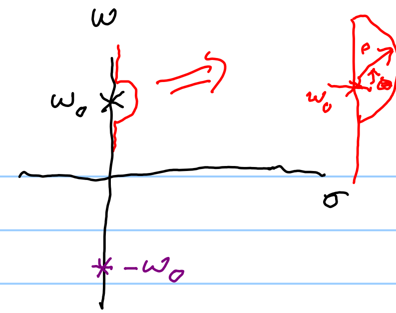
bounded real $S(s)$: S_1 & S_2 bounded real then $S_1 \times S_2$ is bounded real

Lossless PR $y(s) \Rightarrow$ $y(s) + y(-s) = 0$ (scalar) $\Rightarrow y(s) = -y(-s) \Rightarrow$ odd function

(Lossless PR = LPR)

$y(s) = n(s)/d(s)$, n & d real polynomials

Near a pole $y(s)$, PR, is like $y(s) \sim \dfrac{k}{(s-j\omega_0)^m}$
(on $j\omega$ axis @ $j\omega_0$)



$\rho e^{j\theta}$, $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$

to be in $\sigma > 0$

$$= \frac{|k| e^{j\angle k}}{\rho^m e^{j\theta m}}$$

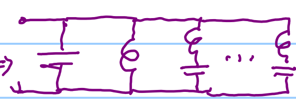
$$= \frac{|k|}{\rho^m} \cdot e^{j(\angle k - m\theta)}$$

$\Rightarrow$ Re $y(s)$ on semi-circle $= \left|\dfrac{k}{\rho^m}\right| \cos(\angle k - m\theta)$

$\Rightarrow \angle k = 0$, $m = 1$ for Re $y(s)$ near the poles in RHP to be ≥ 0

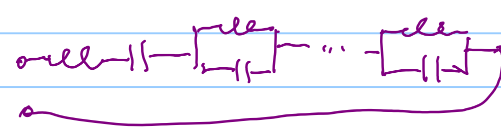
if $y(s)$ is LPR $\quad y(s) = k_\infty s + \dfrac{k_0}{s} + \displaystyle\sum_{i=1}^{n} \left(\frac{k_i}{s - j\omega_i} + \frac{k_i}{s + j\omega_i} \right) = k_\infty s + \frac{k_0}{s} + \sum_{i=1}^{n} \frac{2k_i s}{s^2 + \omega_i^2} = \frac{s\, m_1(s^2)}{d(s^2)} \Rightarrow$ odd

2nd Foster $\Rightarrow$
$y(s) \Rightarrow$



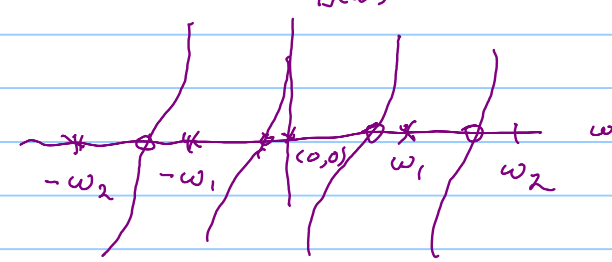
$z(s)$ is also PR & lossless if $y(s)$ is: $z(s) = \hat{k}_\infty s + \dfrac{\hat{k}_0}{s} + \displaystyle\sum_{i=1}^{\hat{n}} \frac{2\hat{k}_i s}{s^2 + \hat{\omega}_i^2}$, $\quad \hat{\omega}_i$ are zeros of $y(s)$ & poles of $z(s)$

1st Foster
$\dfrac{1}{y(s)} = z(s) \Rightarrow$



$$y(j\omega) = k_\infty j\omega + \frac{k_0}{j\omega} + \sum_{i=1}^{n} \frac{2k_i j\omega}{-\omega^2+\omega_i^2} = jB(\omega) = j\left[k_\infty \omega - \frac{k_0}{\omega} + \sum_{i=1}^{n} \frac{2k_i\omega}{-\omega^2+\omega_i^2}\right]$$

$$\frac{dB(\omega)}{d\omega} = k_\infty + \frac{k_0}{\omega^2} + \sum_{i=1}^{n}\left(\frac{2k_i}{-\omega^2+\omega_i^2} + \frac{2k_i\omega(-2\omega)}{-(-\omega^2+\omega_i^2)^2}\right) = k_\infty + \frac{k_0}{\omega^2} + \sum_{i=1}^{n} \frac{2k_i(-\omega^2+\omega_i^2)+4k_i\omega^2}{(-\omega^2+\omega_i^2)^2} \geq 0 \text{ for all } \omega$$



we see the zeros alternate with the poles

Ex:

$$y(s) = \frac{s(s^2+3)}{(s^2+1)(s^2+4)} = \frac{s^3+3s}{(s^4+5s^2+4)} = 0\cdot s + \frac{2k_1 s}{s^2+1} + \frac{2k_2 s}{s^2+4}$$

if were 2 then $y(s)$ is not LPR

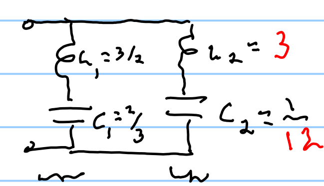
$= 0$ @ $s^2=-1$

$$\frac{s^2+1}{2s}\times\left[\frac{s(s^2+3)}{(s^2+1)(s^2+4)}\right] = \frac{s(s^2+3)}{2s(s^2+4)} = \frac{s^2+3}{2(s^2+4)} = \frac{s^2+1}{2s}\left[\frac{2k_1 s}{s^2+1} + \frac{2k_2 s}{s^2+4}\right] = k_1 + \frac{2k_2}{2}\left(\frac{s^2+1}{s^2+4}\right)$$

$\therefore$ set $s^2=-1$ to get $\left(\frac{s^2+3}{2(s^2+4)}\right)\Big|_{s^2=-1} = \frac{-1+3}{2(-1+4)} = \frac{1}{3} = k_1$, $k_2 = \left(\frac{\cancel{s^2+4}}{\cancel{2s}}\right)\times\left(\frac{\cancel{s}(s^2+3)}{(s^2+1)\cancel{(s^2+4)}}\right)\Big|_{s^2=-4} = \frac{s^2+3}{2(s^2+1)}\Big|_{s^2=-4} = \frac{-1}{-3\times 2} = \frac{1}{3\times 2}$

$\therefore$

$y(s) = \dfrac{\frac{2}{3}s}{s^2+1} + \dfrac{\frac{2}{6}s}{s^2+4} \Rightarrow$



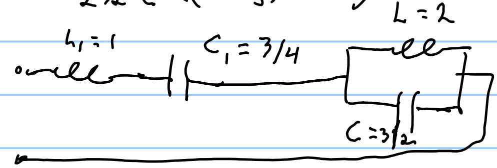
2nd Foster

$z_1 = \dfrac{s^2+1}{\frac{2}{3}s}$ $\quad z_2 = \dfrac{s^2+4}{\frac{1}{3}s}$

both use 4 reactive elements (= minimum numbers)

$z(s) = \dfrac{1}{y(s)} = \dfrac{(s^2+1)(s^2+4)}{s(s^2+3)} = 1\cdot s + \dfrac{4/3}{s} + \dfrac{2\hat{k}_1 s}{s^2+3} = s + \dfrac{4/3}{s} + \dfrac{\frac{2}{3}s}{s^2+3}$

$\hat{k}_1 = \dfrac{s^2+3}{2s}\left(\dfrac{(s^2+4)(s^2+1)}{s(s^2+3)}\right) = \dfrac{(s^2+1)(s^2+4)}{2s^2} \Rightarrow$ set $s^2=-3$ $\quad \hat{k}_1 = \dfrac{(-2)(1)}{2(-3)} = \frac{1}{3}$



1st Foster

$y(s) = \dfrac{s(s^2+3)}{(s^2+1)(s^2+4)} \rightarrow$

$y_1 = \dfrac{s^2+3}{\frac{2}{3}s}$