ENEE610
10/09/18

general 1-port $y(s) = \frac{n(s)}{d(s)}$ design (1st remove non-zero behaviour at ∞, = constants & poles, by long division)

assume

$$y(s) = \frac{n_1 s + n_0}{s^2 + a_1 s + a_0}$$

use companion matrix form

$i(s) = y(s)v(s) = (n_1 s + n_0)\, y_1(s) \cdot v(s)$, $\quad y_1(s) = \frac{1}{s^2 + a_1 s + a_0} \Rightarrow s^2 i_1 + a_1 s i_1 + a_0 i_1 = v$

$y(s) = (n_1 s + n_0) \cdot y_1(s)$

set up state eqs.: $x_1 = i_1$, $x_2 = \dot{x}_1 = \dot{i}_1$, $x_3 = \dot{x}_2 = \ddot{i}_1$; $\dot{} = s = d/dt$

$$s\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} x_2 \\ \ddot{i}_1 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -a_0 & -a_1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix}\begin{bmatrix} v \end{bmatrix}$$

$$\ddot{i}_1 = s^2 i_1 = v - a_1 s i_1 - a_0 i_1 = v - a_1 x_2 - a_0 x_1$$

$$i_1 = \begin{bmatrix} 1 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

but want i as output

$i = n_1 s i_1 + n_0 i_1 = n_1 x_2 + n_0 x_1$

$$i = \begin{bmatrix} n_0 & n_1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$\Rightarrow x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ $\quad sx = Ax + Bv$
$\quad i = Cx$

$C_i \dot{x}_i = i$ capacitor current in capacitor of capacitance C_i if x_i = voltage

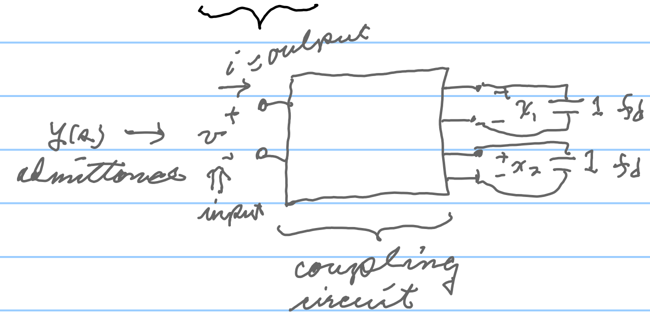
$\therefore$ assume x = capacitor voltage variable, unit capacitor

$\Longrightarrow$ $\dot{x}$ = capacitor currents $= -Ax - Bv$ $\Rightarrow$ $\underbrace{\begin{bmatrix} -B & -A \\ D & C \end{bmatrix}}\begin{bmatrix} v \\ x \end{bmatrix} = \begin{bmatrix} \dot{x} \\ i \end{bmatrix}$

output current $= [Cx] + Dv$

$\therefore$ $\begin{bmatrix} i \\ \dot{x} \end{bmatrix} = \underbrace{\begin{bmatrix} D & C \\ -B & -A \end{bmatrix}}_{\text{constant admittance}} \begin{bmatrix} v \\ x \end{bmatrix}$

$= Y_C$ ← coupling



Numerical example $y(s) = \dfrac{n_1 s + n_0}{s^2 + a_1 s + a_0}$ $\Rightarrow$ $\begin{bmatrix} i \\ \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \left[\begin{array}{c|cc} 0 & n_0 & n_1 \\ \hline 0 & 0 & -1 \\ -1 & a_0 & a_1 \end{array}\right] \begin{bmatrix} v \\ x_1 \\ x_2 \end{bmatrix}$ } voltages on unit capacitors

$Y_C = \begin{bmatrix} 0 & n_0 & n_1 \\ 0 & 0 & -1 \\ -1 & a_0 & a_1 \end{bmatrix}$

$$y(s) = [n_0 \; n_1] \begin{bmatrix} s-0 & -1 \\ a_0 & s-(-a_1) \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

as $y(s)$ = admittance scalar so $y^T(s) = y(s)$

$$= C(sI_2 - A)^{-1}B = \left(C(sI_2-A)^{-1}B\right)^T = B^T(sI_2 - A^T)^{-1}C^T$$

$$T\dot{x} = TAT^{-1}Tx + TBv$$

$$i = CT^{-1}Tx$$

T = non singular constant

let $\hat{x} = Tx$ then $\dot{\hat{x}} = \hat{A}\hat{x} + \hat{B}v$ where $\hat{A} = TAT^{-1}$

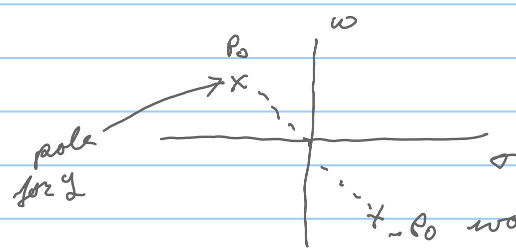
$y = \hat{C} \cdot \hat{x}$ $\qquad \hat{B} = TB, \; \hat{C} = CT^{-1}$

True: If $y(s)$ is PR then a T exists for Y_c to also PR constant $\Rightarrow$ passive

here

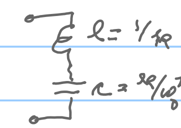
$$\hat{Y}_c = \begin{bmatrix} 1 & 0 \\ 0 & T \end{bmatrix} \begin{bmatrix} D & C \\ -B & -A \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & T^{-1} \end{bmatrix} = \begin{bmatrix} D & \hat{C} \\ -\hat{B} & -\hat{A} \end{bmatrix} = \begin{bmatrix} D & CT^{-1} \\ -TB & -TAT^{-1} \end{bmatrix}$$

Lossless $\qquad y(s) = -\tilde{y}(-s)$



would also be in $y(s)$ but if PR can not be

$\therefore$ all poles are on $j\omega$ axis: $\quad y(s) = \dfrac{ks}{s^2+\omega_0^2}$ a pole @ $s = \pm j\omega_0$ $\quad y(s) = \dfrac{1}{\frac{s^2}{ks} + \frac{\omega_0^2}{ks}} \Rightarrow$



but $\left(\dfrac{ks}{s^2+\omega_0^2}\right)^2$ is not PR $\quad y(s) = \dfrac{1}{s}$ is PR but $\dfrac{1}{s^2}$ is not PR

& k is positive & real

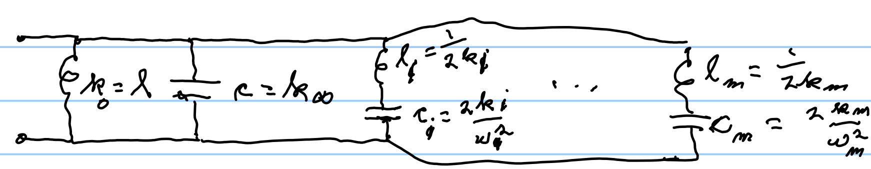
If $y(s)$ is PR then so is $\dfrac{1}{y(s)} = z(s)$

$\therefore$ if $y(s)$ is lossless then so is $z(s) = 1/y(s)$

$\therefore$ make a partial fraction expansion of $y(s) = \dfrac{k_0}{s} + k_\infty s + \sum_{i=1}^{m}\left(\dfrac{k_i}{s - j\omega_i} + \dfrac{k_i^*}{s + j\omega_i}\right)$ if lossless

$y(s) = \dfrac{k_0}{s} + k_\infty s + \sum_{i=1}^{m} \dfrac{2k_i s}{s^2+\omega_i^2}$; $k_i \geq 0$ $\qquad k_i$ = real for PR $y(s)$

+ means parallel



$\Rightarrow$ 2nd Foster form of synthesis