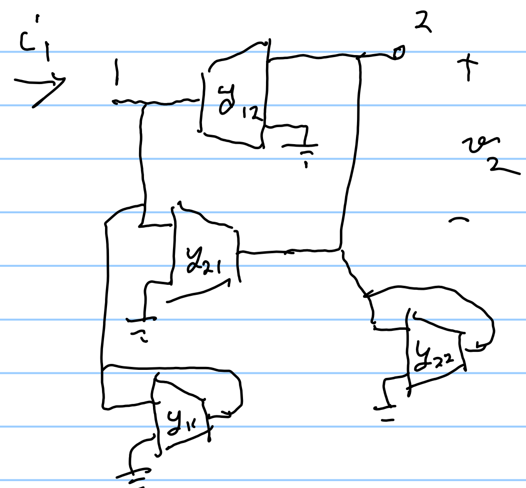
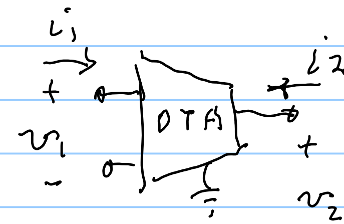
ENEE 610
10/04/18

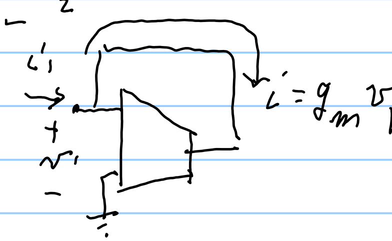
Note Title 10/4/2018

If Y = constant $\quad Y = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix}$, $\quad Y_{OTA} = \begin{bmatrix} 0 & 0 \\ g_m & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = Y_{OTA} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$





$y_{11} = \left.\frac{i_1}{v_1}\right|_{v_2=0}$



can make any constant Y by OTA's ($\Rightarrow$ uses n^2 OTAs) in VLSI

$$Y = Y_{symmetric} + Y_{skew\ symmetric} = \frac{(Y+Y^T) + (Y-Y^T)}{2}; \quad \underset{\underset{\displaystyle Y_{sym}^T}{\shortparallel}}{Y_{sym}} = \frac{Y+Y^T}{2}, \quad \underset{\underbrace{\underset{\displaystyle -Y_{skew}^T}{\shortparallel}}_{\text{gyrators}}}{Y_{skew}} = \frac{Y-Y^T}{2}$$

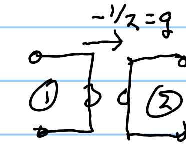
Ex: $Y = \begin{bmatrix} 2 & 4 \\ 5 & 3 \end{bmatrix}$; $Y_{sym} = \frac{1}{2}\left\{\begin{bmatrix} 2 & 4 \\ 5 & 3 \end{bmatrix} + \begin{bmatrix} 2 & 5 \\ 4 & 3 \end{bmatrix}\right\}$

$$= \frac{1}{2}\begin{bmatrix} 4 & 9 \\ 9 & 6 \end{bmatrix} = \begin{bmatrix} 2 & 9/2 \\ 9/2 & 3 \end{bmatrix}$$

$$Y_{skew} = \frac{1}{2}\left\{\begin{bmatrix} 2 & 4 \\ 5 & 3 \end{bmatrix} - \begin{bmatrix} 2 & 5 \\ 4 & 3 \end{bmatrix}\right\}$$

$$= \frac{1}{2}\begin{bmatrix} 0 & -1 \\ +1 & 0 \end{bmatrix} \Rightarrow \text{diagonals} = 0$$

$$= \begin{bmatrix} 0 & -1/2 \\ 1/2 & 0 \end{bmatrix} \Rightarrow$$



$$T^{-1} Y_{sym} T^{-T} = \begin{bmatrix} 1 & 0 \\ -\frac{9}{2}\times\frac{1}{2} & 1 \end{bmatrix}\begin{bmatrix} 2 & 9/2 \\ 9/2 & 3 \end{bmatrix}\begin{bmatrix} 1 & -9/4 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 9/2 \\ 0 & 3-\left(\frac{9}{2}\right)^2\frac{1}{2} \end{bmatrix}\begin{bmatrix} 1 & -9/4 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 0 \\ 0 & \frac{24-81}{8} \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & \frac{-57}{8} \end{bmatrix}$$

$$Y_{sym} = T\begin{bmatrix} 2 & 0 \\ 0 & -\frac{57}{8} \end{bmatrix} T^T \quad , \quad T = \begin{bmatrix} 1 & 0 \\ -9/4 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} 1 & 0 \\ +9/4 & 1 \end{bmatrix}$$

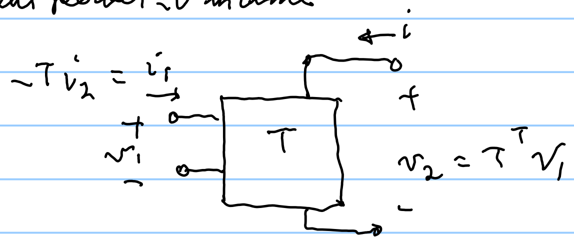
$$= \begin{bmatrix} 1 & 0 \\ 9/4 & 1 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 0 & -\frac{57}{8} \end{bmatrix}\begin{bmatrix} 1 & 9/4 \\ 0 & 1 \end{bmatrix}$$

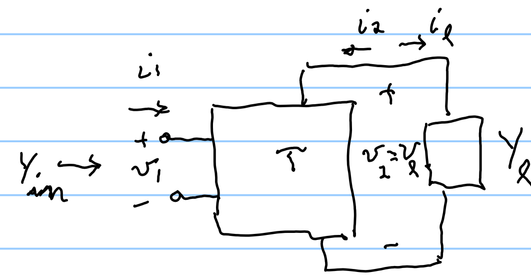
If we have $v_2 = \begin{bmatrix} v_{2_1} \\ v_{2_2} \end{bmatrix} = T^T \begin{bmatrix} v_{1_1} \\ v_{1_2} \end{bmatrix} = T^T v_1$

A transformer has $v^T i = 0$; $v_1^T i_1 + v_2^T i_2 = v_1^T i_1 + (T^T v_1)^T i_2 = v_1^T i_1 + v_1^T T i_2 = 0$ for any v_1

≡ as for any v_1 can cancel $v_1 \Rightarrow i_1 + T i_2 = 0_2 \Rightarrow i_1 = -T i_2$

input power ≡ 0 in time



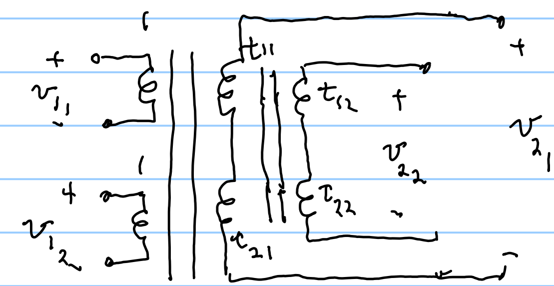


$i_1 = Y_{in} v_1$

$i_\ell = Y_\ell v_\ell = -i_2 = Y_\ell v_2$

$$i_1 = -T i_2 = -T(-Y_\ell v_2) = T Y_\ell T^T v_1 \Rightarrow Y_{in} = T Y_\ell T^T$$

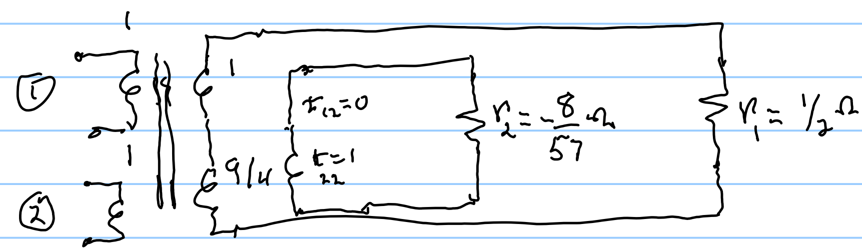
Ex: $Y_e = \begin{bmatrix} 2 & 0 \\ 0 & -\frac{57}{8} \end{bmatrix}$, $T = \begin{bmatrix} 1 & 0 \\ 9/4 & 1 \end{bmatrix}$



$$v_2 = T^T v_1 \Rightarrow v_{2_1} = t_{11} v_{1_1} + t_{21} v_{1_2}$$
$$v_{2_2} = t_{12} v_{1_1} + t_{22} v_{1_2}$$

Ex:

$Y_{in} = \begin{bmatrix} 2 & 9/2 \\ 9/2 & 3 \end{bmatrix} \longrightarrow$



to check if can make with only passive components; look @ Y_{sym} for PR property:

Is $Y = \begin{bmatrix} 2 & 9/2 \\ 9/2 & 3 \end{bmatrix}$ PR? form $\underbrace{\frac{Y+Y^T}{2}}$ and check $x^{T*}\left(\frac{Y+Y^T}{2}\right)x \geq 0$

$$[x_1 \ \ x_2]^* \begin{bmatrix} 2 & 9/2 \\ 9/2 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = [x_1^* 2 + x_2^* 9/2 \ , \ x_1^* 9/2 + x_2^* 3] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = x_1 x_1^* \cdot 2 + x_2^* x_1 \cdot \frac{9}{2} + x_1^* x \cdot 9/2 + x_2^* x_2 \cdot 3$$

This can be negative, try $x_1 = -x_2 = x$, real; $2x^2 - 2\frac{9}{2}x^2 + 3x^2 = x^2(5-9) = -4x^2 \leq 0$

$\therefore Y_{sym}$ here is not PR & can not be made with a passive circuit.

S for an OTA: $Y_{OTA} = \begin{bmatrix} 0 & 0 \\ g_m & 0 \end{bmatrix}$ active (= not passive), $2Y_{sym_{OTA}} = \begin{bmatrix} 0 & g_m \\ g_m & 0 \end{bmatrix}$, $\det Y_{OTA} = \frac{1}{4}(-g_m^2) < 0$

$\Rightarrow$ active as Y_{OTA} is not PR

$$S = (1_n - Y)(1_n + Y)^{-1} = \begin{bmatrix} 1 & 0 \\ -g_m & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ g_m & 1 \end{bmatrix}^{-1} = \begin{bmatrix} 1 & 0 \\ -g_m & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -g_m & 1 \end{bmatrix} \cdot \frac{1}{1}$$

$S_{OTA} = \begin{bmatrix} 1 & 0 \\ -2g_m & 1 \end{bmatrix} \Rightarrow$ can make $S_{inc} = \begin{bmatrix} 1 & 0 \\ -g_{m_1} & 1 \end{bmatrix} S_2 \begin{bmatrix} 1 & -g_{m_2} \\ 0 & 1 \end{bmatrix}$ with also $-g_{m_2} \leq -g_{m_1}$, possible

$\therefore$ can make $S_{in} = T S_x T^T$ if want, but here $S_{OTA} = \begin{bmatrix} 1 & 0 \\ -g_m & 1 \end{bmatrix}$ is active

(as S_{OTA} is not BR since $Y_a = (1_2 - S_a)(1_2 + S_a)^{-1}$ is not PR