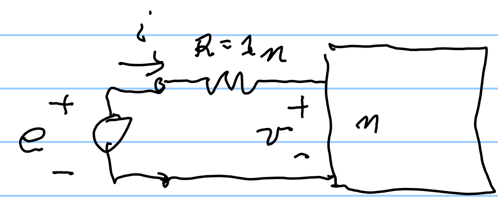
Note Title 10/1/2018

ENEE 610
10/02/18

Hmwk 4 now due 10/09 = Tues.



$p_{in}(t) = v^T(t) i(t) = \mathrm{Re}(v^{T*}(t) i(t))$ for real $v(t), i(t)$

$\mathcal{E}(t) = \int_{-\infty}^{t} p(\tau)\, d\tau \geq 0 \Leftarrow$ for passive

$\Uparrow$

$e \in L_2 \Rightarrow \int_{-\infty}^{\infty} e^{T*}(\tau) e(\tau)\, d\tau < \infty$

$\Rightarrow v^i(t), v^n(t), v(t), i(t)$ are all in L_2

$$\int_{-\infty}^{\infty} v^{T*}(t) i(t)\, dt = \int_{-\infty}^{\infty} V^{T*}(j\omega) I(j\omega)\, df, \quad \omega = 2\pi f$$

$\therefore$ for passive circuits this gives constraints

$S(s)^{n\times n} \Rightarrow$ Bounded Real, $Y(s)^{n\times n} \Rightarrow$ Positive Real $\qquad s = \sigma + j\omega$

a) $S(\sigma)$ is real for $s = \sigma \geq 0$ (real circuits)

b) $S(s)$ is analytic in $\sigma > 0$ (stable)

c) $1_n - S^{T*}(s)S(s)$ is positive semidefinite in $\sigma \geq 0$

(passivity $\Rightarrow$ bounded)

a) $Y(s)$ is real for $s = \sigma > 0$

b) $Y(s)$ is analytic for $\sigma > 0$

c) $Y^{T*}(s) + Y(s)$ is positive semidefinite in $\sigma > 0$ $\Rightarrow V^{T*}\left(Y(s) + Y^{T*}(s)\right)V \geq 0$ in $\sigma > 0$

if rational (BR)

a) coefficients real

b) no poles in $\sigma \geq 0$

c) as for Bounded Real

(PR)

coefficients real

no poles in $\sigma > 0$

as for Positive real

$\frac{n(s)}{d(s)} \Rightarrow$ polynomial n&d

for $n = 1$ (BR) | (PR) | n & d just scalar polynomials

a) real coef. | real coef.

b) only poles (none in $\sigma \geq 0$) (none on $j\omega$ axis) | only poles (none in $\sigma > 0$) (simple poles on $j\omega$ axis) | poles are zeros of the denominator (or ∞)

c) $|S(s)| \leq 1$ in $\sigma \geq 0$
but can use only
$\max |S(j\omega)| \leq 1$ | $Re(Y(-j\omega) + Y(j\omega)) \geq 0$

note $Y^*(s)\big|_{s=j\omega} = Y(-j\omega)$ if PR

Lossless: $\mathcal{E}(\infty) = 0$ $\qquad$ $n \times n$ cases

$1_n - S^{T*}(s)S(s) = 0_n$ $\qquad$ $Re(Y^{T*}(s) + Y(s)) = 0_n$ on $j\omega$ axis

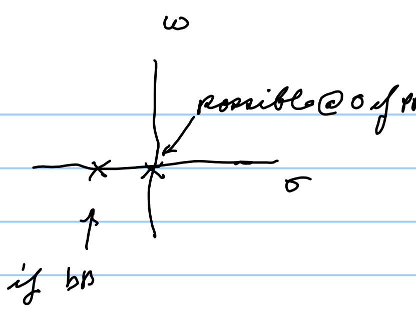
BR, PR $\Rightarrow$ $1_n - S^T(-j\omega)S(j\omega) = 0_n$ $\qquad$ $Re(Y^T(-j\omega) + Y(j\omega)) = 0_n$

let $\omega = s/j$ for an analytic continuation from $s = j\omega$ to all s except poles

$\Rightarrow$ lossless: $1_n - S^T(-s)S(s) = 0_n$ $\qquad$ $Y^T(-s) + Y(s) = 0_n$

$\therefore$ $S^T(-s) = S^{-1}(s)$ $\qquad$ $Y^T(-s) = -Y(s)$

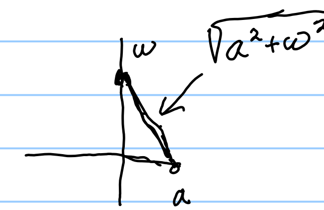
Ex: $F(s) = \dfrac{s+a}{s+b} \Rightarrow$ for br, $\left|\dfrac{a}{b}\right| \le 1$ at $s=0$; at $s=\infty \Rightarrow \dfrac{s}{s} = 1$



pole @ $s = -b \Rightarrow b > 0$ if $F(s)$ is br, if $F(s)$ is pr then $b \geq 0$

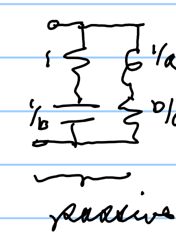
check for br: $\left|\left|\dfrac{s+a}{s+b}\right|\right|_{s=j\omega} \le 1 \Rightarrow \left|\dfrac{j\omega+a}{j\omega+b}\right|^2 = \dfrac{a^2+\omega^2}{b^2+\omega^2} \le 1 \Rightarrow a^2+\omega^2 \le b^2+\omega^2$

$a^2 \le b^2$ but $b > 0$

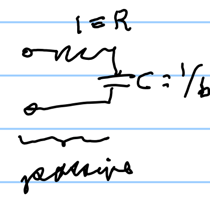


check for pr: (if pr Y, then $Z = 1/Y$ is pr) $\Rightarrow a \geq 0, b \geq 0$

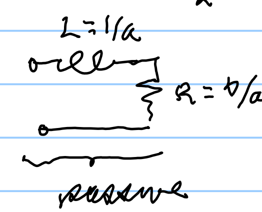
synthesize: $Y(s) = \dfrac{s+a}{s+b} = \dfrac{s}{s+b} + \dfrac{a}{s+b} = \dfrac{1}{\frac{s+b}{s}} + \dfrac{1}{\frac{s+b}{a}} = \dfrac{1}{Z_1} + \dfrac{1}{Z_2} \Rightarrow$



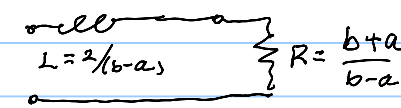
$Z_1 = \dfrac{s+b}{s} = 1 + \dfrac{1}{s/b} \Rightarrow$



, $Z_2 = \dfrac{s+b}{a} \Rightarrow$



$\Rightarrow Y(s) = \dfrac{s+a}{s+b}$ is pr to synthesize $S(\lambda) = \dfrac{\lambda + a}{\lambda + b} \Rightarrow Y(\lambda) = \dfrac{1 - S}{1 + S} = \dfrac{1 - \frac{\lambda + a}{\lambda + b}}{1 + \frac{\lambda + a}{\lambda + b}} = \dfrac{\lambda + b - \lambda - a}{\lambda + b + \lambda + a} = \dfrac{(1-1) + b - a}{2\lambda + (b + a)}$ ← positive

$Y(\lambda) = \dfrac{1}{\frac{2\lambda}{(b-a)} + \frac{b+a}{b-a}} \Rightarrow$ 

if $a = b$, $S = 1 \Rightarrow$ open circuit
$(i = 0)$