Note Title 9/27/2018

ENEE 610
09/27/18

passivity in s-domain use bilateral Laplace transform

$$F(s) = \int_{-\infty}^{\infty} f(t) e^{-st}\, dt \qquad \text{here } s = \sigma + j\omega, \quad j = \sqrt{-1}$$

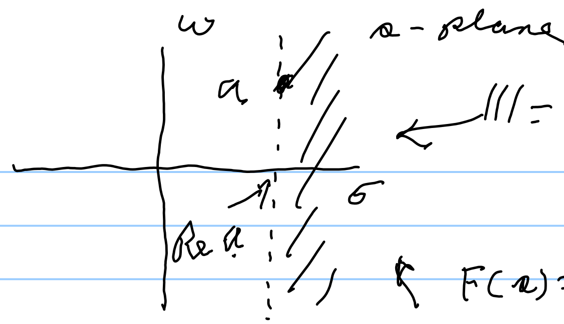
when $s = j\omega$ $\quad F(\omega) = \int_{-\infty}^{\infty} f(t) e^{-j\omega t}\, dt$

Ex: $e^{at}\, 1(t)$, $\quad 1(t) = \begin{cases} 1 & t > 0 \\ 0 & t < 0 \end{cases}$

$$\int_{-\infty}^{\infty} e^{at}\, 1(t)\, e^{-st}\, dt = \int_{0}^{\infty} e^{(a-s)t}\, dt = \frac{1}{(a-s)} \cdot e^{(a-s)t} \Big|_{t=0}^{t=\infty}$$

$$= \frac{-1}{(a-s)} + \frac{1}{(a-s)} e^{(a-s)\cdot\infty}$$

need $e^{+(a-s)\infty} = 0 \Rightarrow Re(a-s) < 0 \Rightarrow Re\, a < Re\, s$



/// = region of convergence of $\int_{-\infty}^{\infty} e^{(a-s)t}\, dt$

$F(s) = \frac{1}{s-a}$

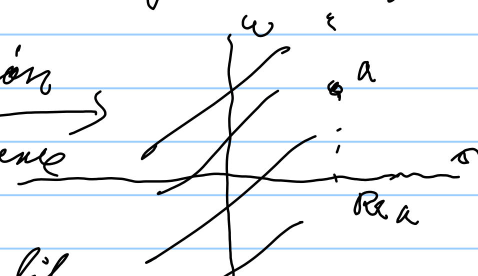
now let s be a complex variable

can use for all s except s = a

Look at $-e^{at}\, 1(-t)$, $\int_{-\infty}^{\infty} -e^{at}\, 1(-t)\, e^{-st}\, dt = \int_{-\infty}^{0} -e^{(a-s)t}\, dt = \frac{-1}{(a-s)} e^{(a-s)t} \Big|_{-\infty}^{0}$

$= \frac{-1}{(a-s)} - \Big(\underbrace{\frac{-1}{(a-s)} e^{(a-s)(-\infty)}}_{} \Big)$ → goes to zero if $Re(a-s) > 0 \Rightarrow Re\, a > Re\, s$

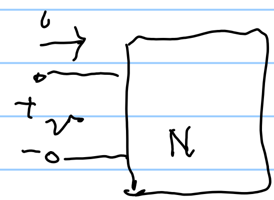
Region of convergence



$F(s) = \frac{1}{s-a}$

∴ I need the region of convergence to identify

$\frac{1}{s-a}$ with a Laplace transform (will assume σ_R in the positive half plane)

power in 

$p_{in}(t) = v^{T*} i$ $\qquad * = j \to -j$

$$= \underbrace{\frac{\left(v^{T*} i + \left(v^{T*} i\right)^{T*}\right)}{2}}_{\uparrow \text{ as } p_{in}(t) \text{ is real}} = \frac{v^{T*} i + i^{T*} v}{2}$$

Parseval's theorem

$$\int_{-\infty}^{\infty} F^{T*}(j\omega)\, G(j\omega)\, df = \int_{-\infty}^{\infty} f^{T*}(t)\, g(t)\, dt$$

if $f(t) = v(t)$, $g(t) = i(t)$ then $\displaystyle\int_{-\infty}^{\infty} v^{T*}(t)\, i(t)\, dt = \int_{-\infty}^{\infty} p(t)\, dt$

If N is passive then $\displaystyle\int_{-\infty}^{\infty} p(t)\, dt \geq 0$

then $\displaystyle\int_{-\infty}^{\infty} V^{T*}(j\omega)\, I(j\omega)\, \frac{d\omega}{2\pi} = \int_{-\infty}^{\infty} V^{T*}(j\omega)\big(Y(j\omega)V(j\omega)\big)\, \frac{d\omega}{2\pi} \geq 0$ for any V if passive

$$= \int_{-\infty}^{\infty} V^{T*}\left[\underbrace{Y(j\omega) + Y^{T*}(j\omega)}_{2}\right] V \geq 0$$ for all V's if passive (V = fixed complex numbers)

$\therefore$ $\underbrace{Y(j\omega) + Y^{T*}(j\omega)}_{2}$ is a positive semidefinite Hermitian form

Ex: If N is an inductor, $Y(s) = \frac{1}{sL}$, $Y(j\omega) = \frac{1}{L(j\omega)}$, $Y^{*T} = \frac{1}{L(-j\omega)} = \frac{-1}{L(j\omega)}$

$\underbrace{Y(j\omega) + Y^{T*}(j\omega)}_{2} = 0_m \Rightarrow \mathcal{E}(\infty) = 0 \Leftarrow$ lossless

$\therefore$ for lossless: $Y(j\omega) + Y^{T*}(j\omega) = 0_n \Rightarrow Y^{T*}(j\omega) = -Y(j\omega)$

if rational $Y^{*}(j\omega) = Y(-j\omega) \Longrightarrow$ $Y(s)$ goes to $Y(-s)$ for $Y^{*}(j\omega)$

$\therefore$ for rational $Y(j\omega) + Y^{T*}(j\omega) = Y(j\omega) + Y^{T}(-j\omega)$

& extend to s-plane by $\omega = s/j \Rightarrow Y(s) + Y^{T}(-s) = 0_n$ for all s except the finite # of poles

Lossless admittance: $Y(s) = -Y^{T}(-s)$

If passive: no poles in $\operatorname{Re} s > 0$ (as stable)

& if real coefficients are all real: $\therefore Y(s)$ is real when s is real, in $\operatorname{Re} s > 0$

Def passive: $Y(s) + Y^{T*}(s)$ is positive semidefinite in $\sigma > 0$

such are called positive real, & PR if rational $\Longleftrightarrow$ can synthesize
with passive
components