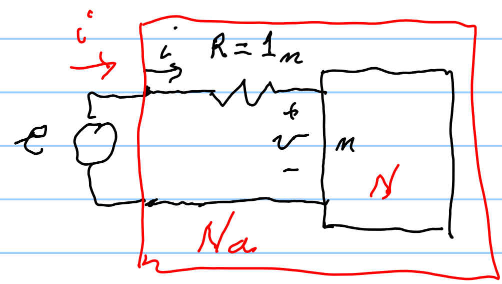
Note Title 9/23/2018

ENEE 610
09/25/18



$i = y_a e \quad , \quad e = v + i$

$v = e - y_a \cdot e = (1_n - y_a) e$

$e = v + i = 2v^i$

$2v^r = v - i \qquad = 2v^r = (1_n - y_a)e - y_a e = (1_n - 2y_a) \cdot 2v^i$

$v^r = S \cdot v^i \, ; \quad S = 1_n - 2y_a$

$Av = Bi$

$\Rightarrow \exists B^{-1} \Rightarrow B^{-1}A v = i \Leftrightarrow Y = B^{-1}A$

$\Rightarrow \exists A^{-1} \Rightarrow v = A^{-1}Bi \Rightarrow Z = A^{-1}B \qquad Z^{-1} = (A^{-1}B)^{-1} = B^{-1}(A^{-1})^{-1} = B^{-1}A$

for $v = v^i + v^r$

$i = v^i - v^r$

$$A \cdot v = A(v^i + v^n) = Bi = B(v^i - v^n) \Rightarrow (A-B)v^i = -(A+B)v^n$$

$$(B+A)^{-1}(B-A)\cdot v^i = v^n \Rightarrow S = (B+A)^{-1}(B-A)$$

if Y exists $= B^{-1}A$ $\Rightarrow S = \left(B(1_n + B^{-1}A)\right)^{-1}\left(B(1_n - B^{-1}A)\right) = (1_n+Y)^{-1}B^{-1}B(1_n - Y)$

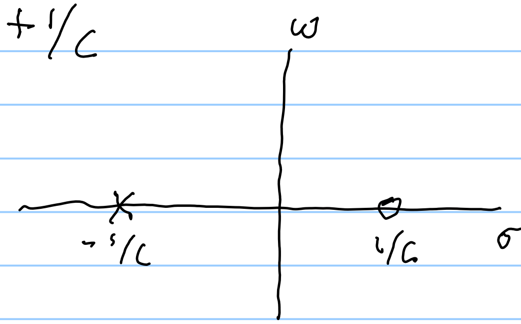
$$= (1_n + Y)^{-1}(1_n - Y)$$

Eg: $n = 1$ = one-port, $Y = sC$ then $S = \left(\frac{1}{1+sC}(1-sC)\right) = \frac{1-sC}{1+sC}$

a pole @ $s = -1/C$ & a zero $s = +1/C$

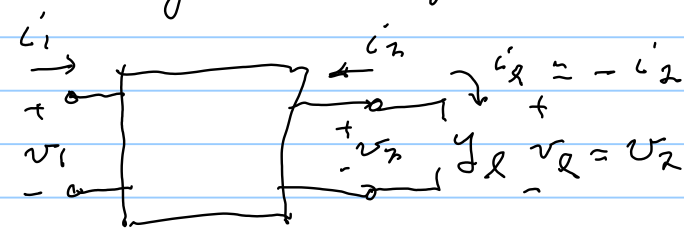
$s = \sigma + j\omega$ plane

here $\left|S(s)\right|\Big|_{s=j\omega} = \frac{|1-j\omega C|}{|1+j\omega C|} = \frac{\sqrt{1+(\omega C)^2}}{\sqrt{1+(\omega C)^2}} = 1$



Look @ the negative impedance converter

$y_{in} = -y_\ell \longrightarrow$



set $v_1 = v_2$

$i_1 = i_2 = -i_\ell = -(y_\ell v_\ell) = -y_\ell v_2$

$$\frac{i_1}{v_1} = y_{in} = \frac{-y_\ell v_2}{v_2} = -y_\ell$$

But hard to build in VLSI

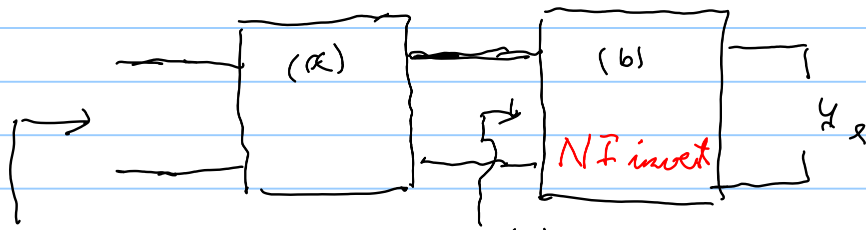
look @ 2 OTAs
$$\begin{cases} i_1 = g_{m_{12}} v_2 \\ i_2 = g_{m_{21}} v_1 \end{cases}$$

set $i_2 = -y_\ell v_2$

$\Rightarrow -y_\ell v_2 = g_{m_{21}} v_1$

$g_{m_{12}} v_2 = i_1$

$$\Rightarrow \frac{-y_\ell v_2}{g_{m_{12}} v_2} = \frac{g_{m_{21}} v_1}{i_1} \Rightarrow \frac{i_1}{v_1} = y_{in} = -\frac{g_{m_{12}} g_{m_{21}}}{y_\ell}$$



NIC = negative impedance converter

$$\frac{-g^{(a)}_{m_{12}} \cdot g^{a}_{m_{21}}}{\left(\frac{-g^{(b)}_{m_{12}}\, g^{(b)}_{m_{21}}}{y_\ell}\right)} \qquad \frac{-g^{(b)}_{m_{12}}\, g^{(b)}_{m_{21}}}{y_\ell}$$

choose for (a) $g^{(a)}_{m_{12}} \approx -g^{(a}_{m_{21}} = g^{(a)}_m \Rightarrow$ gyrator, $g^{(b)}_{m_{12}} = g^{(b)}_{m_{21}} = g^{(b)}_m \Rightarrow$ NI inverts

Then $y_{in} \approx -\dfrac{(g_m^{(a)})^2}{(g_m^{(b)})^2} \cdot y_\ell \Rightarrow$ can build with OTA's

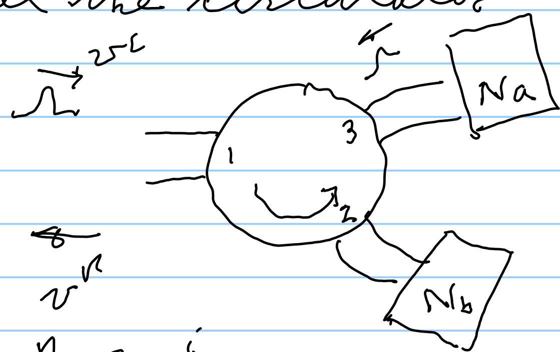
If look @ S for $Y = -y_\ell \Rightarrow S = (1 + (-y_\ell))^{-1}(1 - (-y_\ell))$

$= (1 - y_\ell)^{-1}(1 + y_\ell) = 1/S_\ell$, where $S_\ell = (1+y_\ell)^{-1}(1-y_\ell)$

$\therefore$ can divide S's : $S \Rightarrow S = S_x^{-1}$

of more interest is getting $S = S_a \cdot S_b$ which retains passivity

use the circulator



$v^r = S v^i$

$v_1^r = v_3^i$

$v_2^r = v_1^i$

$v_3^r = v_2^i$

$v_2^r = v_1^i$ = incident on load N_b

v_2^i = reflected from load N_b = $S_b \cdot v_2^r$

$v_3^r = v_2^i \Rightarrow$ into load on port 3 = N_a

v_3^i = reflected voltage from N_a = $S_a v_2^r$

S circulator $\Rightarrow$ $\begin{bmatrix} v_1^r \\ v_2^r \\ v_3^r \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} v_1^i \\ v_2^i \\ v_3^i \end{bmatrix}$; $S_{circulator} = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} = \Sigma$

$\Sigma^T \Sigma = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = 1_3$ $\Rightarrow$ lossless (passive)

then $S_{in} = S_a \cdot S_b$