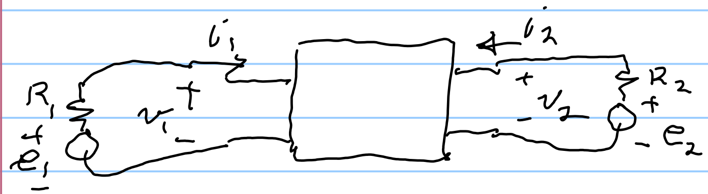
Note Title

ENEE 610
09/20/18

9/20/2018



$v = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$, $i = \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$,

$e = \begin{bmatrix} e_1 \\ e_2 \end{bmatrix} = \begin{bmatrix} R_1 & 0 \\ 0 & R_2 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} + \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$

$i = Yv$, $v = Zi$

normalize to $R_1 = R_2 = 1$

$2v^i = v + i = v + Ri = e \Rightarrow v^i = \frac{1}{2} e$

$\underbrace{2v^i}_{\text{2 incident voltage}} = v + i = e$

$\underbrace{2v^r}_{\text{2 reflected voltage}} = v - i = v - Ri$

$2v^i + 2v^r = v + i + (v - i) = 2v$

$2v^i - 2v^r = v + i - (v - i) = 2i$ $\Leftarrow$

$\Rightarrow \quad v = v^i + v^r$ $\qquad$ sometimes written $a = v^i$
$\quad\;\; i = v^i - v^r$ $\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\quad b = v^r$

S = scattering matrix $\Rightarrow \quad v^r = S v^i$

$$2v^r = v - i = 2S\left(\frac{v+i}{2}\right) = Sv + Si = v - i \Rightarrow Si + i = v - Sv$$

$$\Rightarrow \quad (S + 1_2)\, i = (1_2 - S)\, v \quad \Rightarrow \quad Y = (S+1_2)^{-1}(1 - S)$$

also $\quad (S + 1_2) Y = 1_2 - S \Rightarrow Y - 1_2 = -S - SY = -S(1_2 + Y)$

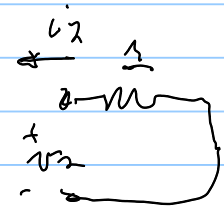
$$\Rightarrow \quad (1_2 - Y)(1_2 + Y)^{-1} = S$$

also $\quad S = (1_2 + Y)^{-1}(1_2 - Y) \quad$ as $\quad (1_2+Y)^{-1}(1_2 - Y) = (1_2 - Y)(1_2 + Y)^{-1}$

$$1_2 - Y^2 = (1_2 - Y)(1_2 + Y) = (1_2 + Y)(1_2 - Y) = 1_2 - Y^2$$

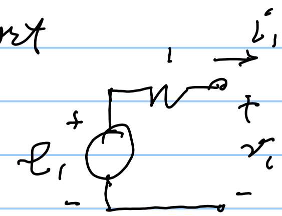
For our 2-port, set $2v_2^i = 0 = e_2 \Rightarrow v_2 + i_2 = 0 \Rightarrow i_2 = -v_2$

2nd port



$2v_2^r = v_2 - i_2 = 2v_2 \Rightarrow v_2 = v_2^r$

1st port



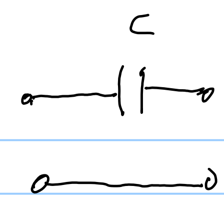
$\Rightarrow e_1 = v_1 + i_1 = 2v_1^i \Rightarrow \left.\dfrac{v_2^r}{v_1^i}\right|_{v_2^i = 0} = S_{21}$

$\frac{1}{2} S_{21} = \frac{v_2}{e_1}$

$$\begin{bmatrix} v_1^r \\ v_2^r \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} v_1^i \\ \underset{0}{\underset{\shortparallel}{v_2^i}} \end{bmatrix} \Rightarrow v_2^r = S_{21} v_1^i \Rightarrow \frac{v_2^r}{v_1^i} = S_{21} = \frac{v_2}{e_1/2} \Rightarrow 2\frac{v_2}{e_1} = S_{21}$$

terminated voltage gain

Ex:



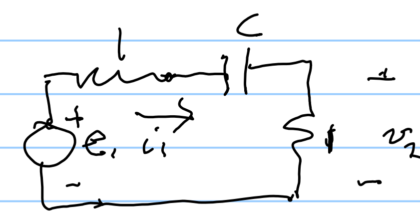
$Y = \begin{bmatrix} sC & -sC \\ -sC & sC \end{bmatrix}$; $\det Y = 0$ no Z exists as Y is singular

$$S = (1_2 + Y)^{-1}(1_2 - Y) = \begin{bmatrix} 1+sC & -sC \\ -sC & 1+sC \end{bmatrix}^{-1} \begin{bmatrix} 1-sC & sC \\ sC & 1-sC \end{bmatrix} = \frac{\begin{bmatrix} 1+sC & sC \\ sC & 1+sC \end{bmatrix}}{1+2sC} \begin{bmatrix} 1-sC & sC \\ sC & 1-sC \end{bmatrix}$$

$$\det(1_2 + Y) = (1+sC)^2 - (sC)^2 = 1 + 2sC$$

$$= \frac{\begin{bmatrix} 1 & 2sC \\ 2sC & 1 \end{bmatrix}}{1+2sC} = S,$$

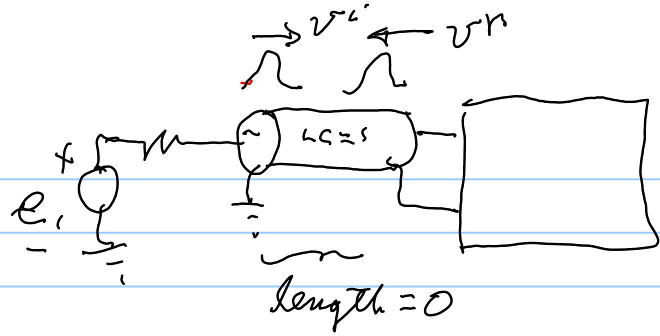
$$S_{21} = \frac{2sC}{1+2sC}$$



$$\left(2 + \frac{1}{sC}\right) i_1 = e_1 \Rightarrow i_1 = \frac{e_1}{2 + 1/sC} = v_2 \Rightarrow \frac{v_2}{e_1} = \frac{1}{2 + 1/sC}$$

as i_1 flows in $R_2 = 1\,\Omega$

$$\frac{v_2}{e_1} = \frac{sC}{1+2sC}$$



gives a physical meaning to S

$$\int_{-\infty}^{T} v^{i\,T} v^i \, d\tau < \infty \quad \text{call } v^i \text{ an } L_2 \text{ function} = \text{square integrable}$$

$$= \int_{-\infty}^{t} (v+i)^T (v+i)\, d\tau = \int_{-\infty}^{t} (v^T v + v^T i + i^T v + i^T i)\, d\tau$$

$$= \int_{-\infty}^{t} v^T v \, d\tau + \int_{-\infty}^{t} i^T i \, d\tau + 2\underbrace{\int_{-\infty}^{t} v^T i \, d\tau}_{} < \infty \quad \text{for all } t$$

(can let $t \Rightarrow \infty$)

$p(t)$ ∴

= power into 2 port

shows if $\mathcal{E}(t) = \int_{-\infty}^{t} p(\tau)\, d\tau > 0$ that v & i are also L_2 functions