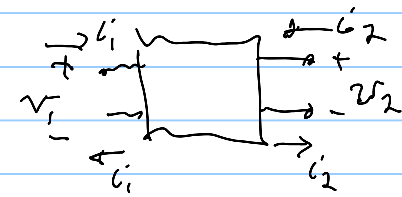
ENEE 610
09/18/18

2-port

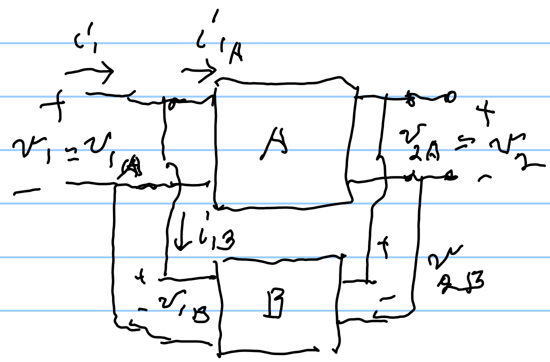


$p_{in}(t) = v_1(t)i_1(t) + v_2(t)i_2(t)$

$v = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}, \quad i = \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$

admittance $\quad i = Yv$

impedance $\quad v = Zi \quad \Rightarrow \quad ZY = 1_2 = YZ$

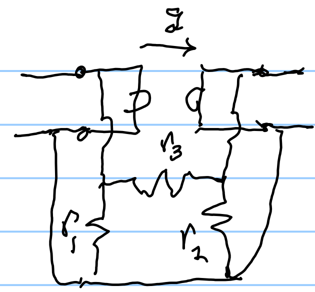


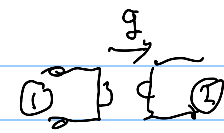
$i_A + i_B = Y_A v_A + Y_B v_B, \quad v_A = v_B = v$

$\quad \| $

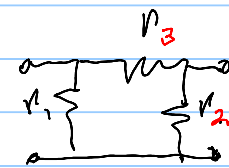
$i = (Y_A + Y_B)v \Rightarrow \quad Y = Y_A + Y_B$

Ex :



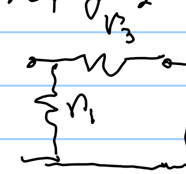


$$Y_{gyr} = \begin{bmatrix} 0 & g \\ -g & 0 \end{bmatrix}$$



$$Y_{res} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \times \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{matrix} y_{11} v_1 + y_{12} v_2 \\ = y_{21} v_1 + y_{22} v_2 \end{matrix}$$

$y_{11} = i_1 / v_1$ if $v_2 = 0$ = short

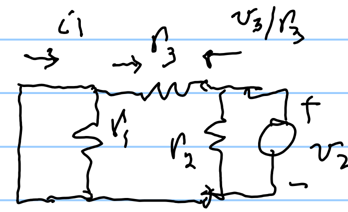
$\Rightarrow$ 

$y_{11} = g_1 + g_3$

$g_1 = 1/R_1, \; g_3 = 1/r_3$

$$y_{22} = \left.\frac{i_2}{v_2}\right|_{v_1=0} = g_2 + g_3 \quad , \quad y_{12} = \left.\frac{i_1}{v_2}\right|_{v_1=0}$$

$$= -g_3$$



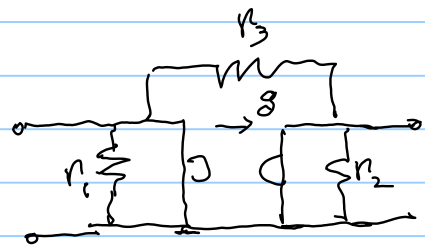
$i_1 = -g_3 v_3$

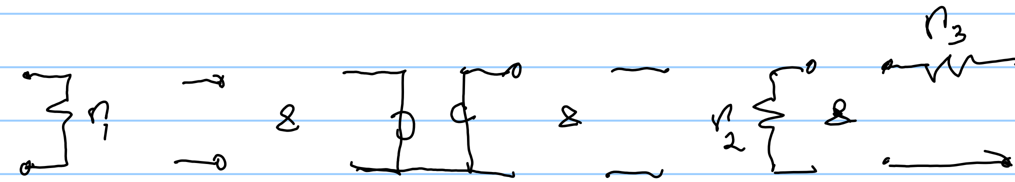
$y_{21} = -g_3$ by symmetry

$$Y_{res} = \begin{bmatrix} g_1 + g_3 & -g_3 \\ -g_3 & g_2 + g_3 \end{bmatrix}$$

$$Y = Y_{gyr} + Y_{res} = \begin{bmatrix} 0 & g \\ -g & 0 \end{bmatrix} + \begin{bmatrix} g_1+g_3 & -g_3 \\ -g_3 & g_2+g_3 \end{bmatrix} = \begin{bmatrix} g_1+g_3 & g-g_3 \\ -g-g_3 & g_2+g_3 \end{bmatrix}$$

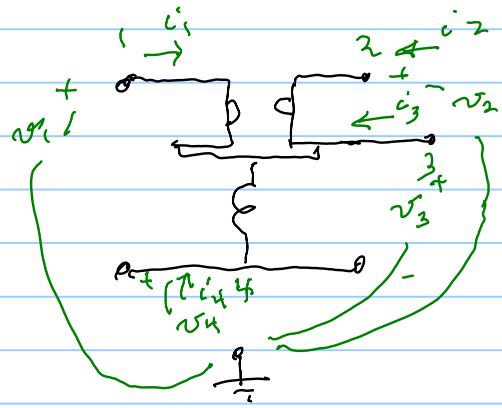
if $g = g_3$ then $Y = \begin{bmatrix} g_1+g_3 & 0 \\ -2g_3 & g_2+g_3 \end{bmatrix}$ $\Rightarrow$ unilateral "gain"
i.e. $i_1 = 0 \cdot v_2$

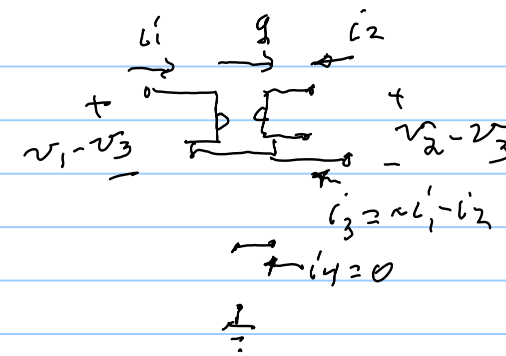




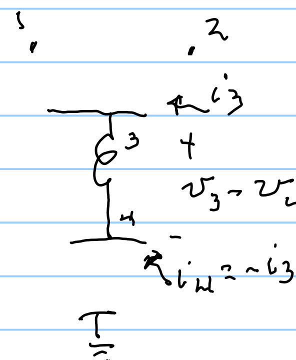
$$= Y = \begin{bmatrix} g_1 & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & g \\ -g & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & g_2 \end{bmatrix} + \begin{bmatrix} g_3 & -g_3 \\ -g_3 & g_3 \end{bmatrix}$$

Indefinite nodal admittance:





$$Y_{gyr} = \begin{bmatrix} 0 & g & -g & 0 \\ -g & 0 & g & 0 \\ g & -g & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$



$$Y_{inductor} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{sL} & -1/sL \\ 0 & 0 & -1/sL & +1/sL \end{bmatrix}$$

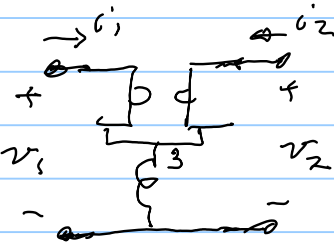
$$Y_{ind} = \begin{bmatrix} 0 & g & -g & 0 \\ -g & 0 & g & 0 \\ g & -g & 1/sL & -1/sL \\ 0 & 0 & -1/sL & 1/sL \end{bmatrix} = \text{indefinite admittance}$$

$$\begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{bmatrix} = Y_{ind} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix}$$

move ground to node 4 $\Rightarrow v_4 = 0$ & ignore i_4 (as $i_4 = -i_1 - i_2 - i_3$)

$$\begin{array}{c} \\ i_{port} \left\{ \right. \\ \\ \end{array} \begin{bmatrix} i_1 \\ i_2 \\ \hline i_3 \end{bmatrix} = \left[\begin{array}{cc|c} 0 & g & -g \\ -g & 0 & g \\ \hline g & -g & 1/a_2 \end{array}\right] \begin{bmatrix} v_1 \\ v_2 \\ \hline v_3 \end{bmatrix} \left.\begin{array}{c} \\ \\ \end{array}\right\} v_{port}, \qquad Y_{def} \text{ nodal admittance} = \begin{bmatrix} 0 & g & -g \\ -g & 0 & g \\ g & -g & 1/a_2 \end{bmatrix}$$

But if want the 2-port Y for



node 3 is internal $\Rightarrow i_3 = 0$

$\therefore$ can solve for v_3 (& eliminate it) in terms of v_1 & v_2

$$0 = \begin{bmatrix} i_{port} \\ \hline i_{int} \end{bmatrix} = \left[\begin{array}{c|c} Y_{11} & Y_{12} \\ \hline Y_{21} & Y_{22} \end{array}\right] \begin{bmatrix} v_{port} \\ \hline v_{int} \end{bmatrix} \Rightarrow 0 = Y_{21} v_{port} + Y_{22} v_{int} \quad \text{solve}$$

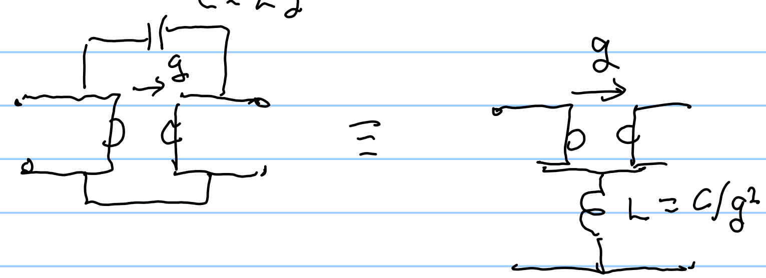
$$v_{int} = -Y_{22}^{-1} Y_{21} v_{port}$$

$$i_{port} = Y_{11} v_{port} - Y_{12} Y_{22}^{-1} Y_{21} v_{port} = \left(Y_{11} - Y_{12} Y_{22}^{-1} Y_{21} \right) v_{port}$$

(2-port) $$Y = Y_{11} - Y_{12} Y_{22}^{-1} Y_{21} \Rightarrow \text{Example} \quad Y = \begin{bmatrix} 0 & g \\ -g & 0 \end{bmatrix} - \begin{bmatrix} -g \\ g \end{bmatrix} \frac{1}{1/sL} \begin{bmatrix} g & -g \end{bmatrix}$$

$$Y = \begin{bmatrix} 0 & g \\ -g & 0 \end{bmatrix} - sL \begin{bmatrix} -g^2 & g^2 \\ g^2 & -g^2 \end{bmatrix} = \begin{bmatrix} sLg^2 & -sLg^2 + g \\ -sLg^2 - g & sLg^2 \end{bmatrix}$$

$C = Lg^2$



if $g = 10^{-3}$

$L = 10^6 \times C$

if $C = 1 \mu F_d$, $L = 1$ Hy