Note Title 9/12/2018

ENEE 610
09/13/18

$E\alpha x = Ax + Bu \Rightarrow (E\alpha - A)x = Bu \Rightarrow x = (E\alpha - A)^{-1}Bu$

(semistate eqs) $y = Cx \Rightarrow y = C(E\alpha - A)^{-1}Bu \Rightarrow T(\alpha) = C(E\alpha - A)^{-1}B$

assume $(\alpha E - A)^{-1}$ exists

svd = singular value decomposition $E = V\Lambda U$, Λ = diagonal of $= \underbrace{[\lambda_1, \dots, \lambda_s, 0, \dots, 0]}_{b}$

V = orthogonal, $b \times b \Rightarrow V^T V = I_b$

U = " " " $\Rightarrow U^T U = I_b$

s = rank of E, $\lambda_i > 0$, $i = 1, \dots, s$

$V\Lambda U\alpha x = Ax + Bu$

multiply by $V^T \Rightarrow \Lambda\alpha\hat{x} = V^T A U^T \hat{x} + V^T Bu$

$y = CU^T\hat{x}$

$\hat{x} = Ux \Rightarrow U^{-1}\hat{x} = x = U^T\hat{x}$

$$\Lambda = \Lambda_s \dotplus 0_{b-s} = \left[\begin{array}{c|c} \begin{matrix}\lambda_1 & & \\ & \ddots & \\ & & \lambda_s\end{matrix} & 0 \\ \hline 0 & 0_{b-s} \end{array}\right]$$

$$\Rightarrow s\begin{bmatrix} \Lambda_\delta & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} \hat{x}_\delta \\ \hat{x}_{b-\delta} \end{bmatrix} = \begin{bmatrix} \hat{A}_{11} & \hat{A}_{12} \\ \hat{A}_{21} & \hat{A}_{22} \end{bmatrix}\begin{bmatrix} \hat{x}_\delta \\ \hat{x}_{b-\delta} \end{bmatrix} + \begin{bmatrix} \hat{B}_1 \\ \hat{B}_2 \end{bmatrix} u \quad ; \quad V^T A U^T = \hat{A}, \; V^T B = \hat{B}$$

$$C U^T = \hat{C}$$

multiply by Λ_δ^{-1}

$$\Rightarrow \begin{bmatrix} 1_\delta & 0 \\ 0 & 0 \end{bmatrix} s\hat{x} = \begin{bmatrix} \Lambda_\delta^{-1}\hat{A}_{11} & \Lambda_\delta^{-1}\hat{A}_{12} \\ \hat{A}_{21} & \hat{A}_{22} \end{bmatrix}\hat{x} + \begin{bmatrix} \Lambda_\delta^{-1}\hat{B}_1 \\ \hat{B}_2 \end{bmatrix} u$$

for last $b-\delta$ rows $\quad \underset{(b-\delta)\times(b-\delta)}{0} \, s\hat{x}_{b-\delta} = \hat{A}_{21}\hat{x}_\delta + \hat{A}_{22}\hat{x}_{b-\delta} + \hat{B}_2 u$

assume that $\hat{A}_{22}^{-1}$ exists; then $\hat{x}_{b-\delta} = -\hat{A}_{22}^{-1}\hat{A}_{21}\hat{x}_\delta - \hat{A}_{22}^{-1}\hat{B}_2 u \quad (b-\delta)$

substitute in 1st δ equations (dynamic ones)

$$s 1_\delta \hat{x}_\delta = \Lambda_\delta^{-1}\hat{A}_{11}\hat{x}_\delta + \Lambda_\delta^{-1}\hat{A}_{12}\left\{\left[-\hat{A}_{22}^{-1}A_{21}\right]\hat{x}_\delta - \hat{A}_{22}^{-1}\hat{B}_2 u\right\} + \Lambda_\delta^{-1}\hat{B}_1 u$$

$$= \mathcal{A}\hat{x}_\delta + \mathcal{B}u$$

$$y = \begin{bmatrix} \hat{C}_1 & \hat{C}_2 \end{bmatrix} \begin{bmatrix} \hat{x}_\delta \\ \hat{x}_{n-\delta} \end{bmatrix} = \hat{C}_1 \hat{x}_\delta + \hat{C}_2 \overset{(n-\delta)}{\downarrow}\left[-\hat{A}_{22}^{-1} \hat{A}_{21} \hat{x}_s - \hat{A}_{22}^{-1} \hat{B}_2 u \right]$$

$$= \mathcal{C} \hat{x}_\delta + \mathcal{D} u$$

now have state variable equations, $\hat{x}_\delta$ = state-vector size δ

$$\frac{dx}{dt} = \mathcal{A} x + \mathcal{B} u \; , \; y = \mathcal{C} x + \mathcal{D} \cdot u \quad (\text{rewriting } x_\delta \text{ as } x)$$

$T(s) = \mathcal{C}(s 1_\delta - \mathcal{A})^{-1} \mathcal{B} + \mathcal{D}$ given $T(s)$ desire to create $\mathcal{A}, \mathcal{B}, \mathcal{C}, \mathcal{D}$ & from that create a circuit

if single input single output (u & y are 1-vectors) then

$T(s) = \frac{N(s)}{D(s)}$, N & D polynomial in s

$$= \frac{b_0 s^\delta + b_1 s^{\delta-1} + \cdots + b_{\delta-1} s + b_\delta}{s^\delta + a_1 s^{\delta-1} + \cdots + a_{\delta-1} s + a_\delta} \Rightarrow d = T(\infty) = b_0$$

let $\hat{T}(s) = T(s) - b_0 = \underbrace{\frac{(b_1 - a_1 b_0) s^{\delta-1} + \cdots + (b_\delta - a_\delta b_0)}{s^\delta + a_1 s^{\delta-1} + \cdots + a_\delta}}$ has $\hat{T}(\infty) = 0$

$$= \mathcal{C}(s\mathbb{1}_\delta - \mathcal{A})^{-1}\mathcal{B}$$

for use of companion matrix for $\mathcal{A}$

$$\mathcal{A} = \begin{bmatrix} 0 & 1 & 0 & \cdots & & 0 \\ 0 & 0 & 1 & 0 & \cdots & 0 \\ \vdots & & & & \ddots & 0 \\ 0 & & & & & 1 \\ -a_\delta & -a_{\delta-1} & \cdots & & & -a_1 \end{bmatrix}, \quad \mathcal{B} = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}$$

$$\mathcal{C} = [b_\delta - a_\delta b_0, \ldots, b_1 - a_1 b_0], \quad \mathcal{D} = b_0$$

Example: $T(s) = z(s) = \dfrac{2s+6}{s^2+8s+12} = \dfrac{2s+6}{(s+2)(s+6)}$; $y(s) = \dfrac{1}{z(s)} = \dfrac{s^2+8s+12}{2s+6}$

for $\mathcal{D}$: $T(\infty) \Rightarrow \dfrac{2s}{s^2} \Rightarrow 0$, $\mathcal{D} = 0$; $\delta = \text{degree den.} = 2$ $\left(@\infty \Rightarrow \dfrac{s^2}{2s} = \dfrac{s}{2}\right)$

$$\mathcal{A} = \begin{bmatrix} 0 & 1 \\ -12 & -8 \end{bmatrix}, \quad \mathcal{B} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad \mathcal{C} = [6 \ \ 2], \quad \mathcal{D} = 0$$

$$y = \mathcal{C}(s1_2 - \mathcal{A})^{-1}\mathcal{B}u, \quad \mathcal{C}(s1_2 - \mathcal{A})^{-1}\mathcal{B} = [6 \ \ 2]\begin{bmatrix} s-0 & -1 \\ 12 & s+8 \end{bmatrix}^{-1}\begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} s & -1 \\ 12 & s+8 \end{bmatrix}^{-1} = \frac{[6 \ \ 2]}{s^2+8s+12}\begin{bmatrix} s+8 & 1 \\ -12 & s \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \frac{[6(s+8)+2(-12), \ 2s+6]}{s^2+8s+12}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \frac{2s+6}{s^2+8s+12}$$