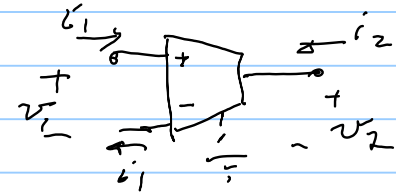
OTA



$i_2 = g_m v_1$

$i_1 = 0$

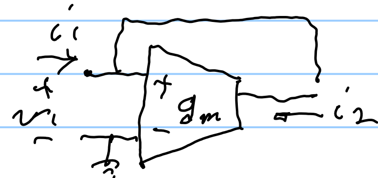
$P(t) = v_1 i_1 + v_2 i_2 = v_2 i_2 = v_1 v_2 \cdot g_m$ can be negative if $v_2 = -v_1 \neq 0$, $g_m > 0$

" power into OTA

as $p(t) < 0$ can occur then the energy in can be negative $\Rightarrow$ active device

$$E(t) = \int_{-\infty}^{t} p(\tau)\,d\tau$$ passive if ≥ 0 for all v & i

But

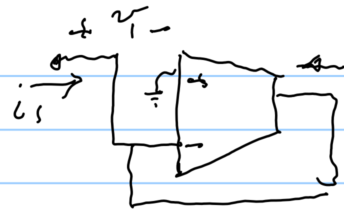


$i_1 = i_2 = g_m v_1$ (equivalent to $\equiv$

$R = 1/g_m$, $G = \frac{1}{R} = g_m$

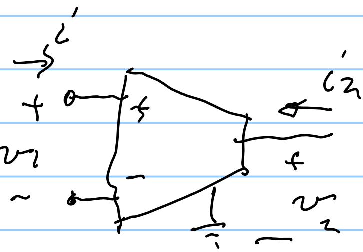
passive, $p = v_1 i_1 = v_1^2 g_m \geq 0$

also



$i_1 = i_2 = -g_m v_1$, $\Longrightarrow$

$R = -\frac{1}{g_m} \Longrightarrow G = -g_m < 0$



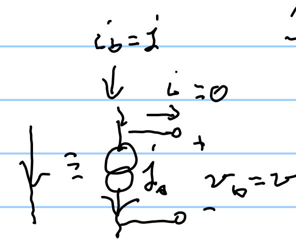
$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ g_m & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \Rightarrow i = Y v, \; Y \text{ is a } 2\times 2 \text{ admittance matrix}$$

In general if every branch has an admittance

$A(A) \cdot v = 1_b \, i$; $v = v_b - v_s$
$i = i_b - j_s$

$1_b = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & 0 \cdots & 0 \\ \cdot & \cdot & \cdots & 1 \end{bmatrix}$ = identity matrix size $b \times b$

note



$$A(s)\, v_b = 1_b\, i_b - 1_b \cdot j_s \qquad v_b = \mathcal{C}^T \cdot v_t \,,\quad i_b = \mathcal{J}^T \cdot i_\ell \Rightarrow \mathcal{C}\mathcal{J}^T = 0_{t,\ell}$$

$$b = t + \ell$$

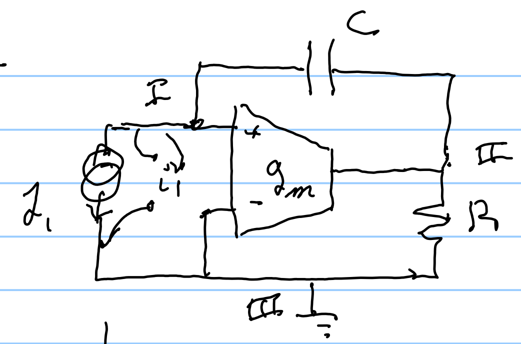
$$A \cdot \mathcal{C}^T v_t - 1_b \mathcal{J}^T i_\ell = -j_s$$

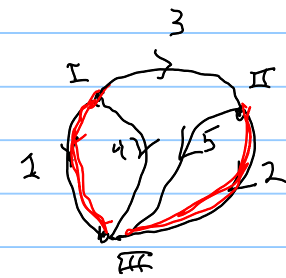
$$\underbrace{\left[A\mathcal{C}^T \,\vdots\, -\mathcal{J}^T \right]}_{b \times (t+\ell)} \underbrace{\begin{bmatrix} v_t \\ i_\ell \end{bmatrix}}_{b \text{ vector}} = \underbrace{-j_s}_{b \text{ vector}} \qquad x = \begin{bmatrix} v_t \\ i_\ell \end{bmatrix}$$

$\times\ \mathcal{C}$ = cut set matrix

$$[\underbrace{\mathcal{C}A\mathcal{C}^T}_{t \times t} \,\vdots\, \underbrace{-\mathcal{C}\mathcal{J}^T}_{t \times \ell}]\, x = -\mathcal{C} j_s \Rightarrow \underbrace{\underbrace{\mathcal{C}A\mathcal{C}^T}_{\text{an admittance}} v_t + 0_{t,\ell} \cdot i_\ell = -\underbrace{\mathcal{C} j_s}_{\text{Norton type current sources}}}_{t \text{ eqs. in } t \text{ unknowns}}$$

Example





$b=5, \; t=2, \; l=3$

$$\begin{bmatrix} 0 \\ 0 \end{bmatrix} = C i_b = \left[\begin{array}{cc|ccc} 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & -1 & 0 & 1 \end{array}\right] \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \end{bmatrix}_b$$

$$\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = \left[\begin{array}{cc|ccc} -1 & 1 & 1 & 0 & 0 \\ -1 & 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 & 1 \end{array}\right] v_b$$

$$j_a = \begin{bmatrix} j_1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad -C j_a = -\begin{bmatrix} 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} j_1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -j_1 \\ 0 \end{bmatrix}$$

$$Av = Bi$$

$$= \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & G & 0 & 0 & 0 \\ 0 & 0 & sC & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & g_m & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \end{bmatrix} = 1_5 i$$

$$\mathcal{C}A\mathcal{C}^T = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & G & 0 & 0 & 0 \\ 0 & 0 & sC & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & g_m & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & -1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & sC & 0 & 0 \\ 0 & G & -sC & g_m & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & -1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} sC & -sC \\ -sC+g_m & sC+G \end{bmatrix}$$

= nodal admittance matrix

$$\mathcal{C}A\mathcal{C}^T v_t = -\mathcal{C} j_s \Rightarrow \begin{bmatrix} sC & -sC \\ -sC+g_m & sC+G \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}_b = \begin{bmatrix} -j_1 \\ 0 \end{bmatrix}$$

$\Rightarrow$ only $t = 2$ rather than $b = 5$ eqs. to solve

$$\det(\mathcal{C}A\mathcal{C}^T) = sC(sC+G) - (-sC)(-sC+g_m) = sC(G+g_m)$$

(1,2) entry of $(\mathcal{C}A\mathcal{C}^T)^{-1}$ is $(-1)^{1+2} \times (2,1)$ minor $= (-1)^3(-sC) = sC$

(2,1) entry $= (-1)^3(-sC+g_m) = sC - g_m$, $(1,1) = sC+G$, $(2,2) = sC$

$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix}_b = \frac{1}{sC(G+g_m)} \begin{bmatrix} sC+G & sC \\ sC-g_m & sC \end{bmatrix} \begin{bmatrix} -j_1 \\ 0 \end{bmatrix} = \begin{bmatrix} -\frac{(sC+G)}{sC(G+g_m)} \cdot j_1 \\ -\frac{(sC-g_m)}{sC(G+g_m)} \cdot j_1 \end{bmatrix}$$

$v_b = C^T v_t$, $i_b = A v$.   works well if have admittances for branches

$\begin{array}{c} v_b \\ \shortparallel \\ v \end{array}$