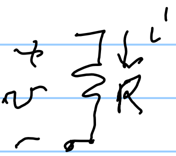
Note Title 9/6/2018

ENEE 610
09/06/18

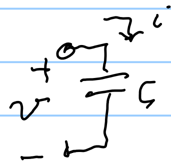
$Av = Bi$ linear if $A$ & $B$ are constants

1-ports



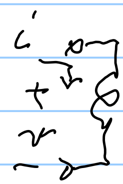
$i = Gv$

or $v = Ri$



$i = C\frac{dv}{dt}$

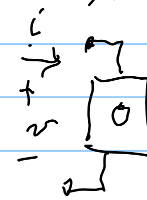
$= Csv$



$v = L\frac{di}{dt}$

$v = Lsi$

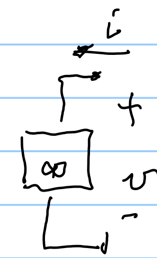
$s = d/dt$



$i = 0$

$v = 0$

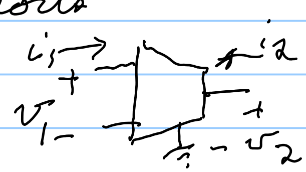
nullator



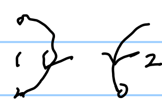
$\Rightarrow 0 \cdot v = 0 \cdot i$

norator

2-ports

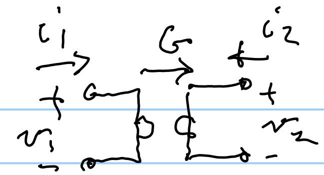


$\Longrightarrow$

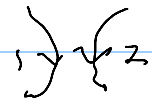


$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ g_m & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$$

OTA = operational transconductance amplifier

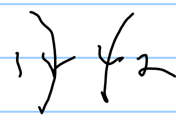



gyrator



$i = \begin{bmatrix} 0 & G \\ -G & 0 \end{bmatrix} v$ , $G = -G^T$ $\quad$ $V^T$ = transpose

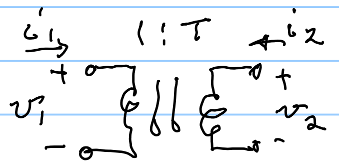

ideal op-amp



$v_1 = 0 \Rightarrow$ virtual connection
$i_1 = 0$

$$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$$

$v_2$ & $i_2$ are free & fixed by external connections




ideal transformer

$v_2 = T v_1$

$p_{in}(t) = 0 = v_1 i_1 + v_2 i_2 = v_1 i_1 + T v_1 i_2 = 0$ $\quad$ as $v_1$ can be anything

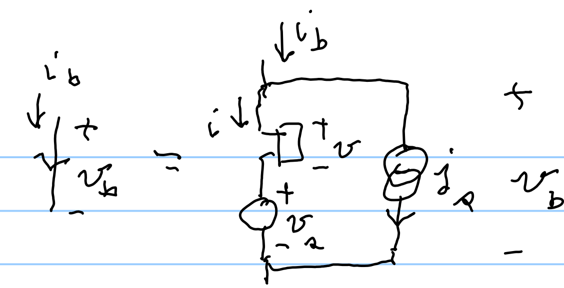
$\Rightarrow i_1 + T i_2 = 0$

$$\begin{bmatrix} -T & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 1 & T \end{bmatrix}\begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$$

$i_1 = -T i_2$

general theory: $\begin{cases} \text{KVL: } v_b = C^T v_t, & O_\ell = \mathcal{J} v_b \\ \text{KCL: } i_b = \mathcal{J}^T i_\ell, & O_t = C i_b \end{cases}$

connection laws




$i_b = i + j_s$, $v_b = v + v_s$ } $i = i_b - j_s$, $v = v_b - v_s$

components, linear time-invariant

$Av = Bi$ ; $A$ & $B$ are $b \times b$

combine $\Rightarrow$ $A(v_b - v_s) = B(i_b - j_s) \Rightarrow Av_b - Bi_b = \underbrace{Av_s - Bj_s}_{\text{sources}}$ (Norton-Thevenin Type)

$\Rightarrow \underbrace{AC^T}_{b \times t} v_t - \underbrace{B\mathcal{J}^T}_{b \times \ell} i_\ell = Av_s - Bj_s$

$\underbrace{\left[ AC^T \;\vdots\; -B\mathcal{J}^T \right]}_{b \times b} \begin{bmatrix} v_t \\ \vdots \\ i_\ell \end{bmatrix} = \left[ E_s + (-Q) \right] = B \begin{bmatrix} v_s \\ i_\ell \end{bmatrix}$

$\therefore \quad E\, dx/dt = \mathcal{A}x + \mathcal{B}u, \qquad y = Cx, \qquad x = \begin{bmatrix} v_t \\ i_\ell \end{bmatrix},$ $u$ = input = $n$-vector
$x$ = semistate $\delta$-vector
$y$ = output = $m$-vector

$Es \cdot x =$

dim of $E = \delta$ = degree ; $(Es - \mathcal{A})x = \mathcal{B}u$

$y = C \cdot (Es - \mathcal{A})^{-1}\mathcal{B} \cdot u$ , $\mathcal{T}(s)$ = transfer function $= C(sE - \mathcal{A})^{-1}\mathcal{B}$

If $E = 1_\delta$ then $x \to$ state

But we have singular $E$, so will need to manipulate to get state equations

$$s x = Ax + Bu \qquad \mathcal{T}(s) = D + C(s1_\delta - A)^{-1}B$$
$$y = Cx + Du$$

$$\mathcal{T}(\infty) = D$$

Look at getting $T(s) = s$

$$\begin{bmatrix} s & 0 \\ 0 & 0 \end{bmatrix} x = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u \quad , \quad y = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$x_1 = -u \, , \quad s x_1 = x_2 \quad , \quad y = x_2 = s x_1 = s(-u)$$

$$y = -su$$

note can get derivatives due to singular $E$

By transforming the semistate can get

$$\begin{bmatrix} I_s & 0 \\ 0 & N \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} A_1 & 0 \\ 0 & A_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} u$$

$\therefore$ find nonsingular matrices $P$ & $Q$ so

$$PEQ = [I_s \oplus N] \, , \quad PAQ = [A_1 \oplus A_2]$$

$$sPEQQ^{-1}x = PAQQ^{-1}x + PBu, \quad y = CQQ^{-1}x$$

$$s\begin{bmatrix} I_\delta & 0 \\ 0 & N \end{bmatrix}\hat{x} = \hat{A}\hat{x} + \hat{B}u, \quad y = \hat{C}\hat{x}, \qquad \hat{A} = PAQ, \ \hat{B} = PB, \ \hat{C} = CQ$$

$$\hat{x} = Q^{-1}x$$

$N$ is nilpotent, $N^i = 0_{(n-\delta)\times(n-\delta)}$, $N^{i-1} \neq 0$