ENEE610
09/04/18

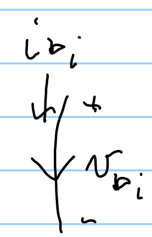
Note Title 9/4/2018

$$v_b = \begin{bmatrix} 1_t \\ K^T \end{bmatrix} v_t = \begin{bmatrix} v_t \\ v_\ell \end{bmatrix} \Rightarrow v_b = \mathcal{C}^T v_t$$

are both known when know K

$$\begin{bmatrix} i_t \\ i_\ell \end{bmatrix} = i_b = \begin{bmatrix} -K \\ 1_\ell \end{bmatrix} i_\ell \Rightarrow i_b = \mathcal{J} i_\ell$$

$KVL \Leftrightarrow KCL$



Power into branch $i = v_{b_i} i_{b_i}$

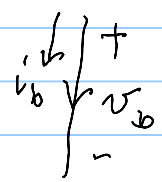
$\therefore$ power into the branches of a graph is $v_b^T i_b(t) = 0$

$$v_t^T \mathcal{C} \cdot \mathcal{J}^T i_\ell = 0$$

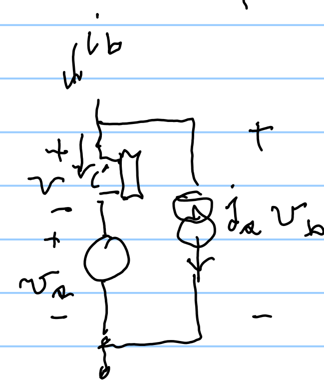
but can ~~put~~ put any voltages in tree branches & any current sources in links

$$\Rightarrow \quad \underset{\substack{\| \\ t\times b}}{\mathcal{C}}\;\underset{\substack{\| \\ b\times \ell}}{\mathcal{J}^T} = 0_{t\times \ell} = \left[1_t \,\vdots\, K_1\right]\begin{bmatrix} -K_2^T \\ 1_\ell \end{bmatrix}^T = 0 = -K_1^{TT} + K_2$$

Circuit equations

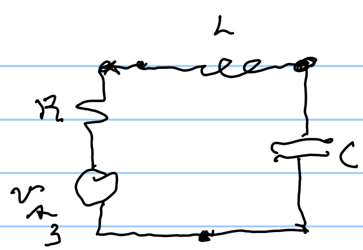


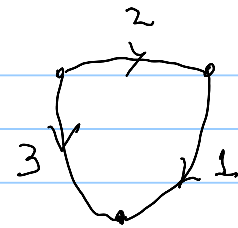
$\Longrightarrow$



$$v_b = v + v_s$$
$$i_b = i + j_s$$

$A v = B i$ for linear components





branch 1: $C\dfrac{dv_1}{dt} = i_1 = C s v_1$

$s = d/dt$

branch2: $L\dfrac{di_2}{dt} = v_2 = sL i_2 = v_2$

branch 3: $v_{b_3} = v_3 + v_s = R i_3 + v_{s_3}$

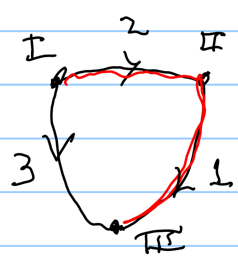
$\Rightarrow v_3 = R i_3 = v_{b_3} - v_{s_3}$

$$\begin{bmatrix} Cs & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ \underset{\uparrow}{v_3} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & sL & 0 \\ 0 & 0 & R_3 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix}$$

$v_{b_3} - v_{s_3}$

$$= \begin{bmatrix} Cs & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} v_{1b} \\ v_{2b} \\ v_{3b} \end{bmatrix} - \begin{bmatrix} Cs & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ v_{s_3} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & sL & 0 \\ 0 & 0 & R_3 \end{bmatrix} i_b$$

$$\begin{bmatrix} C_A & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} e^T v_t - \begin{bmatrix} 1 & 0 & 0 \\ 0 & R_L & 0 \\ 0 & 0 & R_3 \end{bmatrix} \tilde{\mathcal{J}}^T i'_e = \begin{bmatrix} C_A & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ v_{R_3} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ v_{R_3} \end{bmatrix}$$



$\sim$ = Tree :

KCL 1) $0 = 1 \cdot i'_{b_1} + 0 \cdot i'_{b_2} + 1 \cdot i'_{b_3}$ $\Rightarrow e = \left[\begin{array}{cc|c} 1 & 0 & 1 \\ 0 & 1 & 1 \end{array}\right]$

2) $0 = 0 \cdot i'_{b_1} + 1 \cdot i'_{b_2} + 1 \cdot i'_{b_3}$

KVL: 3) $0 = -1 \cdot v_{b_1} - 1 \cdot v_{b_2} + 1 \cdot v_{b_3}$ $\tilde{\mathcal{J}} = \left[\begin{array}{cc|c} -1 & -1 & 1 \end{array}\right]$

$$\begin{bmatrix} C_A & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}_b - \begin{bmatrix} 1 & 0 & 0 \\ 0 & R_L & 0 \\ 0 & 0 & R_3 \end{bmatrix} \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix} \begin{bmatrix} i'_{e_3} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ v_{R_3} \end{bmatrix}$$

$$\Rightarrow \left[\begin{array}{cc|c} C_A & 0 & -1 \\ 0 & 1 & -R_L \\ 1 & 1 & R_3 \end{array}\right] \cdot \begin{bmatrix} v_1 \\ v_2 \\ i'_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ v_{R_3} \end{bmatrix}$$

$\Rightarrow$ b eqs. & b unknowns, $x = \begin{bmatrix} v_t \\ i'_e \end{bmatrix}$

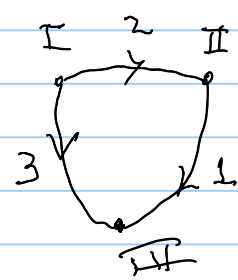
$$\begin{bmatrix} C & 0 & 0 \\ 0 & G & -L \\ 0 & 0 & 0 \end{bmatrix} dx = \begin{bmatrix} 0 & 0 & 1 \\ 0 & -1 & 0 \\ -1 & -1 & -R_3 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} v_{s_3} \Rightarrow E\frac{dx}{dt} = Ax + Bu,$$

u = input = 1 vector here
x = semistate = 3 vector

If the output is the voltage on the capacitor then $v_{out} = v_1$

y = output v_{out} = 1 vector here

$$y = [1 \;\; 0 \;\; 0]\, x = Cx$$



$$\begin{array}{c|ccc} & 1 & 2 & 3 \\ \hline \mathrm{I} & 0 & 1 & 1 \\ \mathrm{II} & 1 & -1 & 0 \\ \mathrm{III} & -1 & 0 & -1 \end{array}$$

Rows I–II: } incidence matrix

Rows I–III: } augmented incidence matrix incidence incidence transpose

$$\begin{bmatrix} 0 & 1 & 1 \\ 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & -1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} \; ; \det \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} = 4 - 1 = 3 = \# \text{ of trees for this graph}$$