ENEE 610
08/30/18

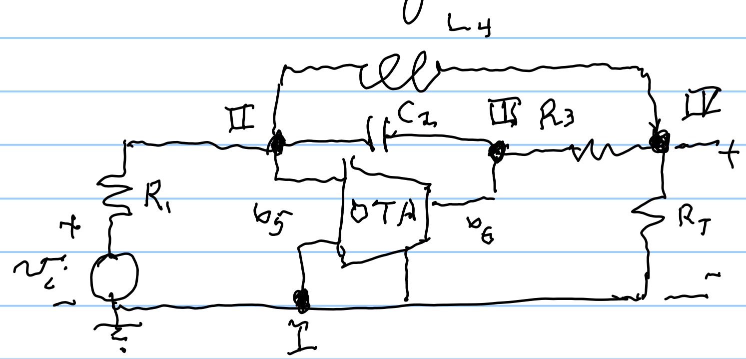
Note Title 8/30/2018

Graphs for circuits

KVL = Σ of voltages around a closed path = 0 $\Rightarrow$ Tie sets
algebraic

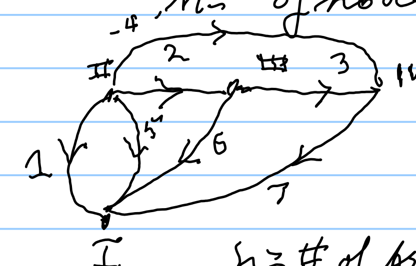
KCL = Σ of currents into a closed surface = 0 $\Rightarrow$ Cut sets
algebraic

Graph = collection of points (= nodes = vertices) &
collection of lines (= branches) connected to nodes

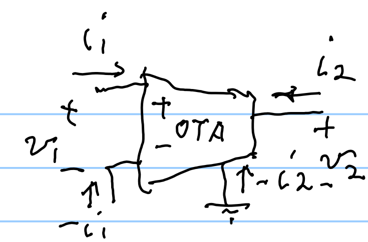


$\Longrightarrow$

n = # of nodes (= 4 here)

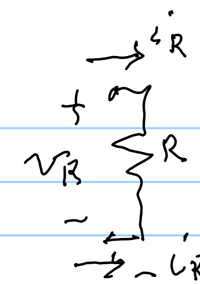


b = # of branches (= 7)



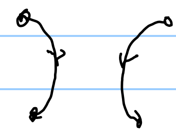
$i_1 = 0$

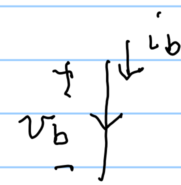
$i_2 = g_m v_1$



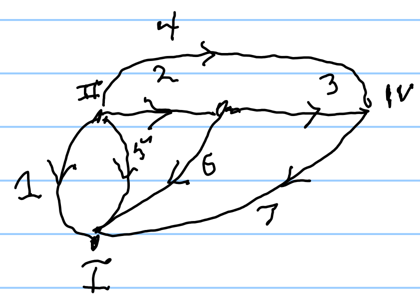
$v_R i_R$ = Power in

$R i_R^2 =$

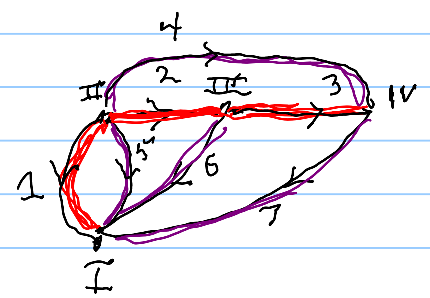




abstract branch

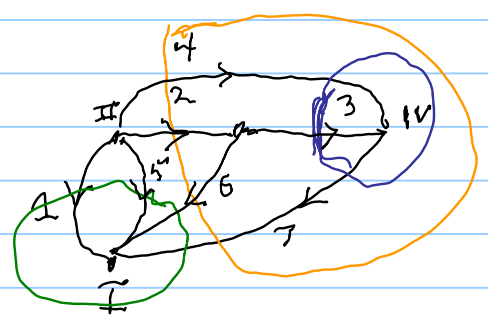


Tree = branches connecting all nodes, no closed path
(# of trees = Prob. 3.12 ⇒ $\det(AA^T)$, incidence matrix



$$v_t = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}, \quad v_b = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ \hline v_4 \\ v_5 \\ v_6 \\ v_7 \end{bmatrix} = \begin{bmatrix} v_t \\ v_\ell \end{bmatrix}$$

$$v_\ell = \begin{bmatrix} v_4 \\ v_5 \\ v_6 \\ v_7 \end{bmatrix}$$



$$i_t = \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix}, \quad i_\ell = \begin{bmatrix} i_4 \\ i_5 \\ i_6 \\ i_7 \end{bmatrix}, \quad i_b = \begin{bmatrix} i_t \\ i_\ell \end{bmatrix}$$

KCL:

$$0 = i_1 + 0 \cdot i_2 + 0 \cdot i_3 + 0 \cdot i_4 + i_5 + i_6 + i_7$$

$$0 = 0 \cdot i_1 + 1 \cdot i_2 + 0 \cdot i_3 + 1 \cdot i_4 + 0 \cdot i_5 - i_6 - i_7$$

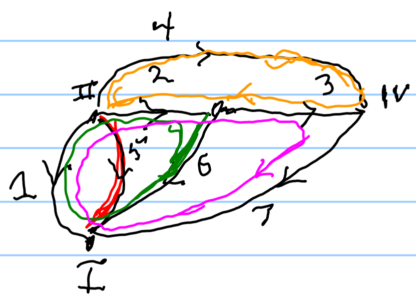
$$0 = 0 \cdot i_1 + 0 \cdot i_2 + 1 \cdot i_3 + 1 \cdot i_4 + 0 \cdot i_5 + 0 \cdot i_6 - i_7$$

$$\Rightarrow \underset{\sim}{0}_3 = \underline{0}_b = C \cdot i_b$$

C = cutset matrix

$$= \left[\underset{\sim}{1}_3 \,\middle|\, K \right]$$

$$\underline{0}_3 = \left[\begin{array}{ccc|cccc} 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & -1 & -1 \\ 0 & 0 & 1 & 1 & 0 & 0 & -1 \end{array}\right] \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \\ i_6 \\ i_7 \end{bmatrix}$$



KVL, tie set 1 = branches 1 & 5

4 $\quad 0 = 0.v_1 - v_2 + 1.v_3 + 1.v_4 + 0.v_5 + 0.v_6 + 0.v_7$

5 $\quad 0 = -1.v_1 + 0.v_2 + 0.v_3 + 0.v_4 + 1.v_5 + 0.v_6 + 0.v_7$ $\quad \Rightarrow \underline{0}_l = \mathcal{T}.v_b$

6 $\quad 0 = -1v_1 + 1.v_2 + 0.v_3 + 0.v_4 + 0.v_5 + 1.v_6 + 0.v_7$

7 $\quad 0 = -1v_1 + 1v_2 + 1v_3 + 0.v_4 + 0.v_5 + 0v_6 + 1.v_7$

$\mathcal{T}$ = tie set matrix

$$\Rightarrow \underline{0}_l = \underbrace{\left[\begin{array}{ccc|cccc} 0 & -1 & 1 & 1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 1 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 & 1 & 0 \\ -1 & 1 & 1 & 0 & 0 & 0 & 1 \end{array}\right]}_{\mathcal{T}} v_b$$

t = # of tree $= n-1$ (for 1 seperate part)

l = # of links (= cotree) $= b-(t) = b-n+1$

$$\mathcal{T} = \left[\begin{array}{c|c} -K^T & 1_l \end{array}\right] \Rightarrow \underline{0} = -K^T v_t + 1_l . v_l \Rightarrow v_l = -(-K^T) v_t$$