

610 Fall 2018 – Homework 5 Due Th 10/18/18

1. (50 points, companion matrix design)

Consider the scalar lossless PR admittance

$$y(s) = [3s(s^2+4)] / [(s^2+1)(s^2+9)]$$

- Explain why this is PR and lossless.
 - Give a state-variable realization in companion matrix form and from it find a (5-port) constant admittance coupling matrix Y_c which when loaded in four capacitors gives $y(s)$ at the remaining port.
 - Discuss passivity of any circuit realizing Y_c .
2. (40 points, PR & BR)
- For the following functions state which are PR and/or BR and where there is a parameter a , for which values of a the function is PR and/or BR.
- $f_1(s) = [s^2 + (2-a)s + 1] / [s^2 + s + a]$.
 - $f_2(s) = [(s^2+1)(s^2+4)] / [s(s^2+2)(s^2+3)]$
 - $f_3(s) = 1/f_1(s)$
 - $f_4(s) = [s^2 - as + 4] / [s^2 + as + 4]$

3. (10 points, tan and tanh)

The partial fraction expansions for $\tan(s)$ and $\tanh(s)$ are

$$\tan(s) = \sum_{(n=1 \text{ to } \infty)} [8s / [(2n-1)^2\pi^2 - 4s^2]]$$

$$\tanh(s) = \sum_{(n=1 \text{ to } \infty)} [8s / [(2n-1)^2\pi^2 + 4s^2]]$$

- Discuss why one is positive-real and the other not.
- The infinite product expansions for \sin and \cos are

$$\sin(\pi s) = \pi s \prod_{(n=1 \text{ to } \infty)} (1 - (s/n)^2)$$

$$\cos(\pi s) = \prod_{(n=1 \text{ to } \infty)} (1 - (2s / \{2n-1\})^2)$$

From these form the infinite product expansion for $\tan(s) = \sin(s) / \cos(s)$ and infer the infinite product expansion for $\tanh(s) = \sinh(s) / \cosh(s)$.