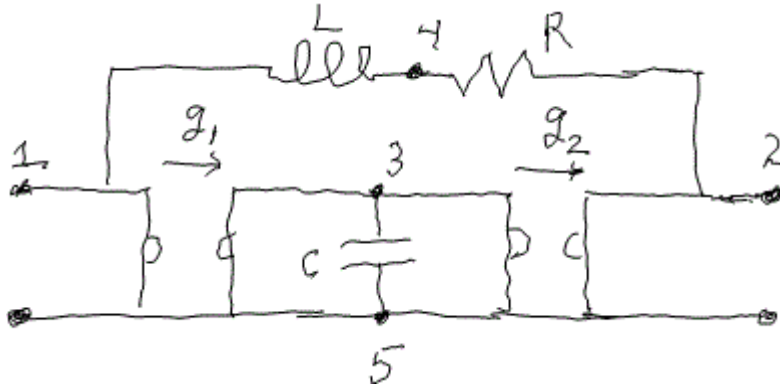


1. (50 points, Indefinite admittance & admittance matrix)



For the above circuit:

- Using the node numbers and a ground off of the circuit, give the 5x5 indefinite admittance matrix,  $Y_{ind}$ .
- Assume the ground is moved to node 5, find the 2-port admittance matrix  $Y(s)$ , having port 1 between nodes 1 & 5 and port 2 between nodes 2 & 5, by
  - eliminating the internal nodes 3 & 4 one at a time (that is, using two different 1x1 matrices  $Y_{nodal22}$ ).
  - eliminating the two internal nodes together (that is, using  $Y_{nodal22}$  as a 2x2 matrix once).
 [give the results such that all four entries have the same polynomial denominator]
- Compare the two procedures.

2. (50 points, state variable for continued fraction)

[this problem uses ideas from Chyi Hwang, Tong-Yi Guo and Leang-San Shieh, "A Canonical State-space Representation for SISO Systems Using Multipoint Jordan CFE," Journal of the Franklin Institute, Vol. 328, No. 2/3, 1991, pp. 207-216.]

For the continued fraction form of the state variable equations  $Esx = Ax + Bu$ ,  $y = Cx$  [the above paper gives these as  $Ksz = Hz + du$ ,  $y = d^T z$ , so here  $K = E$ ,  $H = A$ ,  $d = B$ ,  $z = x$ ] where [see equations (7) of the above paper]

$$E := \begin{pmatrix} k1 & 1 \\ 1 & -k2 \end{pmatrix}$$

$$A := \begin{pmatrix} -h1 & b1 \\ -a1 & -h2 \end{pmatrix}$$

$$B := \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

- a) Show that this form allows the transfer function  $T(s)=B^T(sE-A)^{-1}B$  to be written as the continued fraction

$$\underline{\underline{T}}(s) := \frac{1}{a \cdot s + b + \frac{(s - e)(s - f)}{c \cdot s + d}}$$

Give a, b, c, d, e, f in terms of the entries in E, A, B.

- b) Compare with equation (2) of the above paper.  
c) Use this to give a state space realization for the continued fraction form of the previous impedance.

$$z(s) = (2s+6)/[(s+2)(s+6)]$$