610 final solutions F2016

#1 a) $0_t = \mathcal{C} i'_b \Rightarrow \mathcal{C} = \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & -1 \\ 0 & 1 & 0 & -1 & 1 & 0 \\ 0 & 0 & 1 & 0 & -1 & 1 \end{array}\right]$, $t = 3$, $b = 6$

$0_\ell = \mathcal{J} v_b \Rightarrow \mathcal{J} = \left[\begin{array}{ccc|ccc} -1 & 1 & 0 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 & 1 & 0 \\ 1 & 0 & -1 & 0 & 0 & 1 \end{array}\right]$ $\quad \ell = 3$

b) $i' = Y_{b\times b} v \Rightarrow Y_{6\times 6} = \begin{bmatrix} sC & 0 & 0 & 0 & 0 & 0 \\ 0 & sC & 0 & 0 & 0 & 0 \\ 0 & 0 & sC & 0 & 0 & 0 \\ 0 & 0 & 0 & G & 0 & 0 \\ 0 & 0 & 0 & 0 & G & 0 \\ 0 & 0 & 0 & 0 & 0 & G \end{bmatrix}$

c) $i'_s = [-u, 0\ 0\ 0\ 0\ 0]^T$ for 

$= -i'_{s_1}$ as i'_s directs as i'_1 so $i'_{b_1} = i'_1 + i'_{s_1}$

$x = \text{semi-state} = \begin{bmatrix} v_t \\ i'_\ell \end{bmatrix}$; $v_b = \mathcal{C}^T v_t$, $i'_b = \mathcal{J}^T i'_\ell = i' + i'_s$

$i' = \mathcal{J}^T i'_\ell - i'_s = Y_{b\times b} \mathcal{C}^T v_t \Rightarrow -i'_s = i' - \mathcal{J}^T i'_\ell = Y_{b\times b}\mathcal{C}^T v_t - \mathcal{J}^T i'_\ell = [Y_{b\times b}\mathcal{C}^T \mid -\mathcal{J}^T] \begin{bmatrix} v_t \\ i'_\ell \end{bmatrix}$

$\Rightarrow [Y_{b\times b}\mathcal{C}^T \mid -\mathcal{J}^T] x = -i'_s = [1, 0, 0, 0, 0, 0]^T u$

$= \left\{ \begin{bmatrix} sC & & & & & \\ & sC & & & & \\ & & sC & & & \\ & & & G & & \\ & & & & G & \\ & & & & & G \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix}, \begin{bmatrix} -1 & 0 & 1 \\ 1 & -1 & 0 \\ 0 & 1 & -1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \right\} x = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} u$, $v_{out} = v_1 = [1\ 0\ 0\ 0\ 0\ 0] x$

$= \left[\begin{array}{ccc|ccc} sC & 0 & 0 & -1 & 0 & 1 \\ 0 & sC & 0 & 1 & -1 & 0 \\ 0 & 0 & sC & 0 & 1 & -1 \\ G & -G & 0 & 1 & 0 & 0 \\ 0 & G & -G & 0 & 1 & 0 \\ -G & 0 & G & 0 & 0 & 1 \end{array}\right] x = Bu \Rightarrow \begin{bmatrix} C & 0 & 0 & 0 & 0 & 0 \\ 0 & C & 0 & 0 & 0 & 0 \\ 0 & 0 & C & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \frac{dx}{dt} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 & -1 & 1 \\ -G & G & 0 & -1 & 0 & 0 \\ 0 & -G & G & 0 & -1 & 0 \\ G & 0 & -G & 0 & 0 & -1 \end{bmatrix} x + \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} u$

$y = [1\ 0\ 0\ 0\ 0\ 0] x$

d) $z(s) = v/i'$ with i' into node I & v @ node I w.r.t node IV
$= y/u$ of above eqs. written as $Esx = Ax + Bu$, $y = Cx$ then $y = C(Es - A)^{-1} B u$
But $C(Es-A)^{-1}B = [1, 0 \dots 0](Es - A)^{-1} \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} = (1,1)$ entry of $(Es - A)^{-1}$

so we only need the (1,1) entry
of $(Es-A)^{-1}$ which is Δ_{11}/Δ = (the 1,1 cofactor divided by the determinant
so we calculate Δ_{11} & Δ by any means and $z(s) = \Delta_{11}/\Delta$.

#2. Since this is odd it will be lossless PR if poles & zeros alternate

a) on the $j\omega$ axis $\Rightarrow$ $\underline{\underline{a \geq 4}}$

b) 1st Fosters $\Rightarrow$ expand via partial fraction expansion of Z

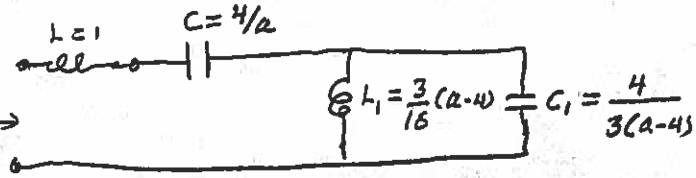
If $a \neq 4 \Rightarrow Z(s) = k_\infty s + \frac{k_0}{s} + \frac{k_1 s}{s^2+4} = \frac{1}{s}(s^2+1)\cdot\frac{1}{s^2+4}\cdot s^2+a$

$$k_\infty = \frac{s^4}{s^3} = 1, \quad k_0 = sZ(s)\Big|_{s=0} = 1\cdot\frac{1}{4}\cdot a = a/4$$

$$k_1 = \frac{s^2+4}{s}Z(s)\Big|_{s^2=-4} = \frac{1}{s^2}(s^2+1)(s^2+a)\Big|_{s^2=-4} = \frac{1}{-4}(-3)(a-4) = \frac{3}{4}(a-4)$$

$$\Rightarrow Z(s) = s + \frac{a/4}{s} + \frac{\frac{3}{4}(a-4)\cdot s}{s^2+4} = s + \frac{1}{\frac{4}{a}s} + \frac{1}{\frac{4}{3(a-4)}s + \frac{1}{\frac{3}{16}(a-4)s}}$$

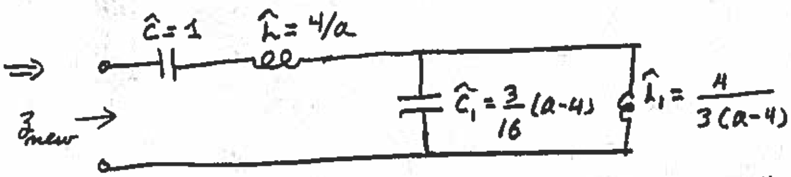
$\Rightarrow$



(if $a=4$, $L_1 = 0 \Rightarrow$ short, $C_1 = \infty \Rightarrow$ short)

c) $Ls \Rightarrow \frac{1}{Ls} = \frac{1}{\hat{C}s}$, $\quad Cs \Rightarrow \frac{1}{Cs} = \frac{1}{\hat{L}s}$

$\Rightarrow$



$\frac{3}{16} = 0.1875$

$4/3 = 1.333\ldots$

$$Z_{new} = \frac{1}{s} + \frac{4}{a}s + \frac{1}{\frac{3}{16}(a-4)s + \frac{1}{\frac{4}{3(a-4)}s}} = \frac{1}{s} + \frac{4}{a}s + \frac{\frac{4}{3(a-4)}s}{\frac{1}{4}s^2+1}$$

$$= \frac{1}{s} + \frac{4}{a}s + \frac{\frac{16}{3(a-4)}\cdot s}{s^2+4}$$

$$= \frac{(s^2+4)}{s(s^2+4)} + \frac{\frac{4}{a}s^2(s^2+4)}{s(s^2+4)} + \frac{\frac{16}{3(a-4)}s^2}{s(s^2+4)} = \frac{\frac{4}{a}s^4 + \left[1+\frac{16}{a}+\frac{16}{3(a-4)}\right]s^2+4}{s(s^2+4}$$

*3. $S_x^T = \frac{dT/dx}{T/x}$; $\frac{dz}{da} = \frac{(s^2+1)\cdot da/da}{s(s^2+4)} = \frac{(s^2+1)}{s(s^2+4)}$

$$S_a^z = \frac{(s^2+1)}{s(s^2+4)} \cdot \frac{1}{\frac{(s^2+1)(s^2+a)}{s(s^2+4)\cdot a}} = \frac{a}{s^2+a}$$

#4, as $y = 1/z$ is degree 4 its cascade uses 4 sections