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ENEE610 HW 5

1.

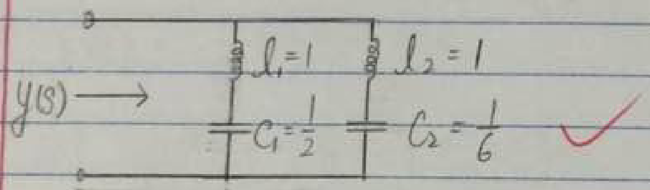
a) $$y(s) = \frac{2s(s^2+4)}{(s^2+2)(s^2+6)} = \frac{2k_2 s}{s^2+2} + \frac{2k_4 s}{s^2+6}$$

$$\frac{s^2+2}{s} y(s)\Big|_{s^2=-2} = \frac{2\times 2}{4} = 0 + 2k_2 + 0, \quad k_2 = \frac{1}{2}$$

$$\frac{s^2+6}{s} y(s)\Big|_{s^2=-6} = \frac{2\times(-2)}{-4} = 0 + 0 + 2k_4, \quad k_4 = \frac{1}{2}$$

$$\Rightarrow y(s) = \frac{s}{s^2+2} + \frac{s}{s^2+6} = \frac{1}{1\cdot s + \frac{1}{\frac{1}{2}s}} + \frac{1}{1\cdot s + \frac{1}{\frac{1}{6}s}}$$

2nd Foster

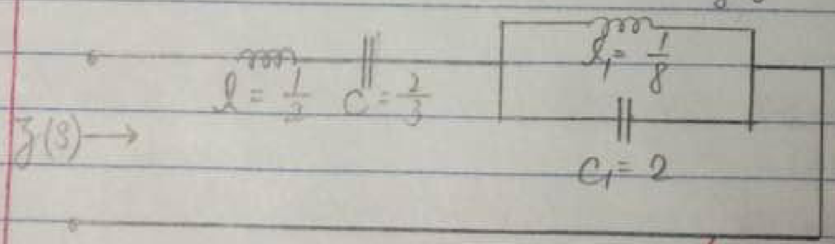


$$z(s) = \frac{(s^2+2)(s^2+6)}{2s(s^2+4)} = k_\infty s + \frac{k_0}{s} + \frac{2k_2 s}{s^2+4}$$

$$\frac{(s^2+4)}{s} z(s)\Big|_{s^2=-4} = \frac{(-2)\times 2}{-8} = 2k_2, \quad k_2 = \frac{1}{4}$$

$$s\, z(s)\Big|_{s=0} = \frac{2\times 6}{2\times 4} = k_0 = \frac{3}{2} \Rightarrow k_\infty = \frac{1}{2}$$

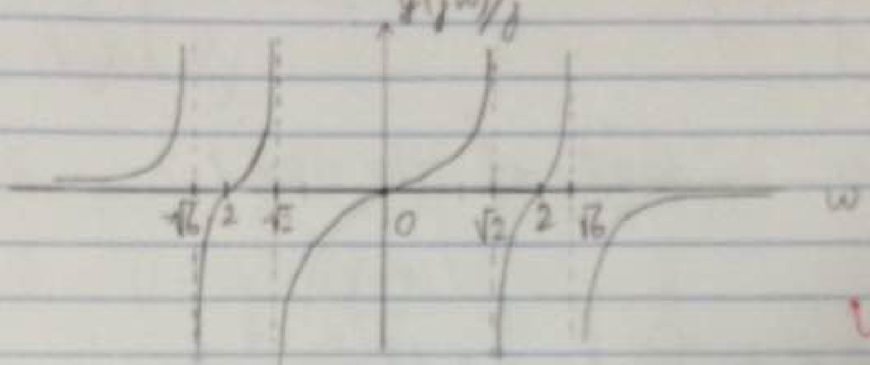
$$\Rightarrow z(s) = \frac{1}{2}s + \frac{1}{\frac{2}{3}s} + \frac{1}{2s + \frac{1}{\frac{1}{8}s}}$$



1st Foster

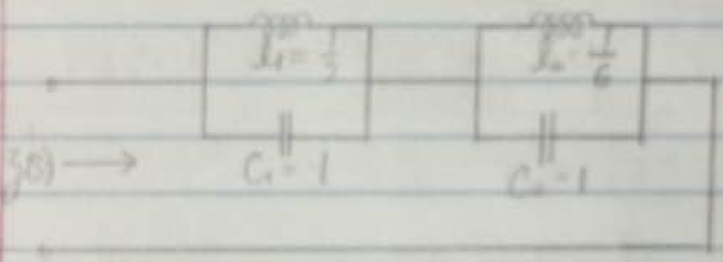
c)

$$\frac{y(j\omega)}{j} = \frac{2j\omega(-\omega^2+4)}{j(-\omega^2+2)(-\omega^2+6)} = \frac{2\omega(-\omega^2+4)}{(-\omega^2+2)(-\omega^2+6)}$$

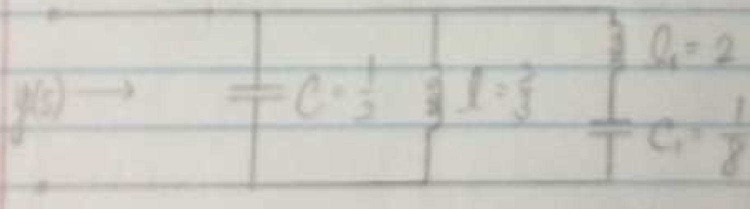


(d) $z(s) = \frac{2s(s^2+4)}{(s^2+2)(s^2+6)}$

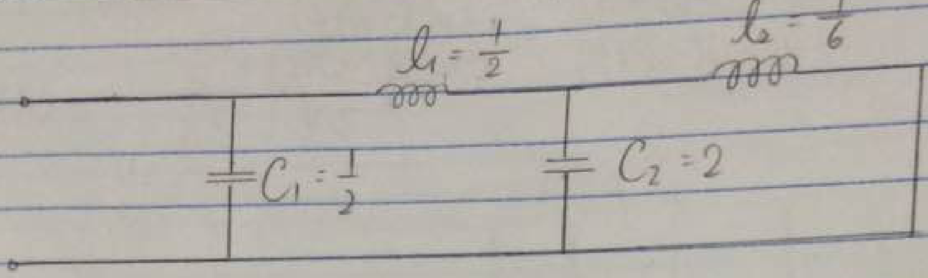
1st Foster dual of 2nd



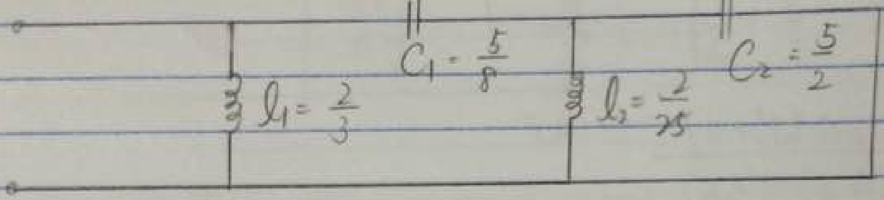
2nd Foster dual of 1st



1st Cauer: dual of the 1st



2nd Cauer: dual of the 2nd



e) Richard's function. $k = 2$

$y(k) = y(2) = \frac{2\times2+8}{6\times10} = \frac{8}{15}$

$R_y = \frac{k y(k) - s y(s)}{k y(s) - s y(k)} = \frac{\frac{16}{15} - \frac{2s^2(s^2+4)}{(s^2+2)(s^2+6)}}{\frac{4s(s^2+4)}{(s^2+2)(s^2+6)} - \frac{8}{15}s} = \frac{\frac{16}{15}(s^4+8s^2+12) - 2(s^4+4s^2)}{4(s^3+4s) - \frac{8}{15}(s^5+8s^3+12s)}$

$= \frac{7s^4 - 4s^2 - 96}{4s^5+2s^3-72s} = \frac{(7s^2+24)(s^2-4)}{2s(s^2-4)(2s^2+9)} = \frac{7s^2+24}{2s(2s^2+9)}$

$y_L = R_y \; y(k)$

2.

a) $S(s) = (1+y)^{-1}(1-y)$

$$= \frac{s^4+8s^2+12}{s^4+2s^3+8s^2+8s+12} \times \frac{s^4-2s^3+8s^2-8s+12}{s^4+8s^2+12} = \frac{s^4-2s^3+8s^2-8s+12}{s^4+2s^3+8s^2+8s+12}$$

✓

include R_y:

$$y_L = R_y \cdot y(k) = \frac{8^4}{15} \cdot \frac{7s^2+24}{2s(2s^2+9)}$$

$$S_L = (1+y_L)^{-1}(1-y_L)$$

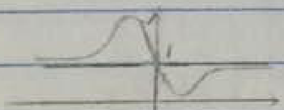
$$= \frac{15s(2s^2+9)}{30s^3+28s^2+135s+96} \times \frac{30s^3-28s^2+135s-96}{15s(2s^2+9)}$$

$$= \frac{30s^3-28s^2+135s-96}{30s^3+28s^2+135s+96}$$ ✓

b) $|S(s)|^2 \leq 1$ in $\sigma > 0$, let $s = j\omega + \sigma$

$$|S(s)|^2 = S(s)\,S(-s) = \frac{s^4-2s^3+8s^2-8s+12}{s^4+2s^3+8s^2+8s+12} \times \frac{s^4+2s^3+8s^2+8s+12}{s^4-2s^3+8s^2-8s+12}$$

$$= 1 \leq 1 \Rightarrow \underline{BR}$$ ✓

c) $S(s)$ is 

has to use imaginary part for poles and zeros.

zeros: $0.715 \pm j1.460$, $0.285 \pm j2.112$
poles: $-0.715 \pm j1.460$, $-0.285 \pm j2.112$

$$y(s) = \frac{2s(s^2+4)}{(s^2+2)(s^2+6)}$$ zeros: 0, $\pm j2$ poles: $\pm j\sqrt{2}$, $\pm j\sqrt{6}$ There is a 0 fo[r] $y(s)$'s zero

$$R_y(s) = \frac{(7s^2+24)(s^2-4)}{2s(s^2-4)(2s^2+9)} = \frac{7s^2+24}{2s(2s^2+9)}$$ zeros: $\pm j\frac{2\sqrt{42}}{7}$ poles: 0, $\pm j\frac{3\sqrt{2}}{2}$ There is a 0 f[or] $R_y(s)$'s pole

✓

3 Hurwitz polynomial

All poles and zeros are in the left half plane or on the imaginary axis

a) $Ev\,P(s) = 2s^6 + 0s^4 + 4s^2 + 2$

$Od\,P(s) = 3s^5 + 4s^3 + 3s$

$$\frac{2}{3}s$$

$$3s^5+4s^3+3s \,\overline{)\, 2s^6 + 0s^4 + 4s^2 + 2}$$

$$2s^6 + \frac{8}{3}s^4 + 2s^2$$

$$-\frac{9}{8}s \rightarrow \text{negati}$$

$$-\frac{8}{3}s^4 + 2s^2 + 2 \,\overline{)\, 3s^5 + 4s^3 + 3s}$$

$\Rightarrow P(s)$ is not Hurwitz ✓

(b) $$e^s = \sum_{n=0}^{\infty} \frac{s^n}{n!}$$

$$= 1 + s + \frac{s^2}{2!} + \frac{s^3}{3!} + \frac{s^4}{4!} + \cdots$$

truncation: $1 + s + \frac{1}{2}s^2 + \frac{1}{6}s^3 + \frac{1}{24}s^4$

$$\frac{1}{4}s$$

$$\frac{1}{6}s^3 + s \,\overline{)\, \frac{1}{24}s^4 + \frac{1}{2}s^2 + 1}$$

$$\frac{1}{24}s^4 + \frac{1}{4}s^2$$

$$\frac{2}{3}s$$

$$\frac{1}{4}s^2 + 1 \,\overline{)\, \frac{1}{6}s^3 + s}$$

$$\frac{1}{6}s^3 + \frac{2}{3}s$$

$$\frac{3}{4}s$$

$$\frac{1}{3}s \,\overline{)\, \frac{1}{4}s^2 + 1}$$

$$\frac{1}{4}s^2$$

$$\frac{1}{3}s$$

$$1 \,\overline{)\, \frac{1}{3}s}$$

$$\frac{1}{3}s$$

$$0$$

all positive

$\Rightarrow$ is Hurwitz ✓

graph for 2.(C)