ASSIGNMENT-4

SIDDHARTH TYAGI ENEE-610: NETWORK THEORY

1.

(a) Condition for Positive-Real:-

- No poles and zeroes in RHP:-

$$\therefore f_a(s) = \frac{s-a}{s+b} \Rightarrow \boxed{a \le 0;\ b \ge 0.}$$

In general; if $s = \sigma + j\omega$; $f_a(s)$ is PR means that

$Re[f_a(s)] > 0$ when $\sigma > 0$.

$$\therefore f_a(\sigma + j\omega) = \frac{(\sigma - a) + j\omega}{(\sigma + b) + j\omega} = \frac{\{(\sigma - a) + j\omega\}\{(\sigma + b) - j\omega\}}{(\sigma + b)^2 + \omega^2}$$

$$\Rightarrow Re[f_a(s)] = \frac{\{\sigma^2 + \omega^2 + \sigma(b-a) - ab\}}{(\sigma + b)^2 + \omega^2}$$

clearly; when $\begin{matrix} a \le 0 \\ b \ge 0 \end{matrix}$ is $\boxed{b \ge a}$; $\boxed{ab \le 0.}$

$\Rightarrow$ all terms of $Re[f_a(s)] \ge 0$.

$\therefore$ Only Condition: $\boxed{\begin{matrix} a \le 0 \\ b \ge 0 \end{matrix}}$.

- Conditions for Bounded-Real :-

- $f_a^*(s) = f_a(s^*)$ is clearly true

- No-singularities in RHP :-

$\therefore \boxed{b \geq 0}$

- $1 - f_a^*(s) f_a(s) > 0$ for all $\sigma > 0$ $\{s = \sigma + j\omega\}$.

$$\cancel{(\sigma - a)^*} \frac{(\sigma - a + j\omega)^* (\sigma - a + j\omega)}{(\sigma + b + j\omega)^* (\sigma + b + j\omega)} = \frac{(\sigma - a)^2 + \omega^2}{(\sigma + b)^2 + \omega^2} \leq 1.$$

$\therefore \quad (\sigma - a)^2 \leq (\sigma + b)^2$

$\therefore \boxed{\{2\sigma + (b - a)\}\{b + a\} > 0} \quad \forall \sigma > 0$

clearly; $\sigma > 0$.

$\therefore \boxed{b + a > 0}$ ✓

Positive-Real:-

$$f_6(s) = \left(\frac{s^2+s+a}{s^2+s+b}\right)$$

- Since, a, b are real $\Rightarrow Y^*(s) = Y(s^*)$
- No poles and zeroes in RHP:-

$$\left.\begin{aligned} z_{1,2} &= -1 \pm \sqrt{1-4a} \\ p_{1,2} &= -1 \pm \sqrt{1-4b} \end{aligned}\right\}$$

clearly, if $\underline{a>0}$

$\Rightarrow \sqrt{1-4a}$ can either be real or imaginary

if imaginary $\Rightarrow \underline{Re(Z_{1,2}) = -1}$

if real $\Rightarrow Re(Z_{1,2}) = -1+\sqrt{1-4a} < 0$ is ensured.

$\Rightarrow \boxed{a>0 \text{ and } b>0}$ will ensure no pole-zero in RHP.

- $\underline{Re(f_6(j\omega)) > 0}$; σ

$$f_6(j\omega) = \frac{(a-\omega^2)+j\omega}{(b-\omega^2)+j\omega} = \frac{\{(a-\omega^2)+j\omega\}\{(b-\omega^2)-j\omega\}}{(b-\omega^2)^2+\omega^2}$$

$$\therefore Re\{f_6(j\omega)\} = (a-\omega^2)(b-\omega^2)+\omega^2$$

$$= \omega^4 + \{1-a-b\}\omega^2 + ab > 0$$

$\cancel{\Rightarrow (1-a-b)^2 - 4ab > < 0.}$ $\cancel{\Rightarrow a^2+b^2-2a-2b+1>0.}$

$\cancel{1+a^2+b^2-2a-2b+2ab-4ab>0}$

This means:-

$$\boxed{(1-a-b)^2 - 4ab < 0}$$ must be true.

Bounded-Real:-

- Since a, b are real $f_b^*(s) = f_b(s^*)$.

- For no singularities in RHP

$$\boxed{b>0}$$

- ~~$I - S^*(j\omega)S$~~ $1 - f_b^*(j\omega) f_b(j\omega) \geq 0$.

clearly: $f_b^*(j\omega) f_b(j\omega) = \dfrac{(a-\omega^2)+j\omega}{(b-\omega^2)+j\omega} \cdot \dfrac{\{(a-\omega^2)-j\omega\}}{\{(b-\omega^2)-j\omega\}}$

$$= \frac{(a-\omega^2)^2+\omega^2}{(b-\omega^2)^2+\omega^2} = \frac{\omega^4+(1-2a)\omega^2+a^2}{\omega^4+(1-2b)\omega^2+b^2}.$$

$\therefore \quad 1 - f_b^*(j\omega) f_b(j\omega) \geq 0$

$\Rightarrow (1-2b)\omega^2 + b^2 - (1-2a)\omega^2 - a^2 \geq 0$

$\therefore \Rightarrow (b+a+2\omega^2)(b-a) \geq 0.$

clearly $\boxed{b-a \geq 0}$ is ~~a~~ possible condition.

$$f_c(s) = \frac{(s^2+as+1)}{(s^2+bs+1)}.$$

- No poles/zeroes in RHP:-

$-a \pm \sqrt{(a^2-4)} \geq 0$

$-b \pm \sqrt{b^2-4} \geq 0.$

clearly; if $a \geq 0$.

$\Rightarrow$ if $a^2-4>0 \Rightarrow -a+\sqrt{a^2-4} \leq 0$.

$<0 \Rightarrow -a \leq 0$

$\Rightarrow$ always Real (pole) ≤ 0

$\therefore \boxed{a \geq 0;\ b \geq 0}$

- $Re[f_c(j\omega)] > 0$;

$$f_c(j\omega) = \frac{(1-\omega^2)+ja\omega}{(1-\omega^2)+jb\omega} = \frac{\{(1-\omega^2)+ja\omega\}\{(1-\omega^2)-jb\omega\}}{(1-\omega^2)^2+(b\omega)^2}$$

$\therefore Re[f_c(j\omega)] = (1-\omega^2)^2 + ab\omega^2$. clearly; $a \geq 0$, $b \geq 0$ ensures this.

$\Rightarrow$ Only condition: $\boxed{a \geq 0;\ b \geq 0}$ for Positive-Real.

✓

For Bounded-Real:-

again $b \geq 0$.

also; $1 - f_c^*(j\omega) f_c(j\omega)$

$$= 1 - \left\{\frac{(1-\omega^2) + ja\omega}{(1-\omega^2) + jb\omega}\right\} \cdot \left\{\frac{(1-\omega^2) - ja\omega}{(1-\omega^2) - jb\omega}\right\}.$$

$$= 1 - \frac{(1-\omega^2) + a^2\omega^2}{(1-\omega^2)^2 + b^2\omega^2}$$

$$= \frac{(b^2 - a^2)\omega^2}{(1-\omega^2)^2 + b^2\omega^2} > 0$$

$$\therefore \quad \boxed{b^2 - a^2 > 0}$$

or $\boxed{|b| > |a|}$

Positive-Real:-

$$f_d(s) = \frac{as}{(s^2+b)^b}.$$

clearly, for PR $\Rightarrow$ relative degree b/w ~~..~~ numerator and denominator must be max. 1. $0 \le b \le 1$

$$\Rightarrow \boxed{b = 0 \text{ or } 1}.$$

(a) When $b=0$

$$\Rightarrow f_d(s) = as$$ which will be $\boxed{\text{+ve real if } a>0}$.

(b) When $b=1$.

$$f_d(s) = \frac{as}{(s^2+1)} \quad ; \quad s = \sigma + j\omega$$

$$= a \frac{(\sigma + j\omega)}{(\sigma^2 - \omega^2 + 1) + j2\sigma\omega}$$

$$= a \frac{(\sigma + j\omega)(\sigma^2 - \omega^2 + 1 - j2\sigma\omega)}{(\sigma^2 - \omega^2 + 1)^2 + 4\sigma^2\omega^2}$$

$$\Rightarrow Re[f_d(s)] = a[\sigma^3 + \sigma + \sigma\omega^2] \ge 0 \Rightarrow \boxed{a \ge 0}$$

$\therefore$ Only condition of PR on a is $\boxed{a>0}$.

· Bounded-Real:-

$$f_d(s) = \frac{as}{(s^2+1)^b}$$

~~again if b>1 ⇒ the initial-state response will.~~
~~blows and~~

when $\underline{b=1}$:-

$$f_d(j\omega) = \frac{a \times j\omega}{1-\omega^2}.$$

$$1 - f_d(j\omega)\, f_d^*(j\omega)$$

$$= 1 - \frac{a^2\omega^2}{(1-\omega^2)^2} \quad \alpha \geq 0.$$

$$\therefore (1-\omega^2)^2 - a^2\omega^2 \geq 0.$$

$$\therefore \boxed{a^2\omega^2 \leq (1-\omega^2)}$$ must be satisfied for B.R.

let $g(\omega) = (1-\omega^2)^2 - a^2\omega^2$

let $x = \omega^2 \geq 0 \Rightarrow g(x) = (1-x)^2 - a^2x = x^2 - (a^2+2)x + 1 \geq 0.$

At $x = \frac{a^2+2}{2}$ $g(x)$ minimum. $g(x)_{min} = \left(\frac{a^2+2}{2}\right)^2 - (a^2+2)\cdot\frac{a^2+2}{2} + 1$

$$= 1 - \frac{(a^2+2)^2}{4} \geq 0$$

$$\Rightarrow (a^2+2)^2 \leq 4.$$

$$\Rightarrow -4 \leq a^2 \leq 0.$$

$$\Rightarrow a = 0$$

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(d) $f_d = \frac{as}{(s^2+1)^b} = \frac{N(s)}{D(s)}$

PR: ① $Re[f_d(j\omega)] \geq 0$ for all $0 \leq \omega \leq \infty$
② The degrees of $N(s)$ and $D(s)$ differ at most by 1
(from the textbook)

$Re[f_d(j\omega)] = 0$, from ② $\Rightarrow$ if $a \neq 0$, $0 \leq b \leq 1$
$\underline{a = 0,\ b \text{ arbitrary for PR}}$

BR: $|f_d(s)|^2 = \left(\frac{a\omega}{(1-\omega^2)^b}\right)^2 = \frac{a^2\omega^2}{(1-\omega^2)^{2b}} \leq 1$

$\Rightarrow (1-\omega^2)^{2b} - a^2\omega^2 \geq 0 \Rightarrow -a^2\omega^2 \geq 0$
$-a^2 \geq 0 \Rightarrow \underline{a = 0 \text{ for BR}}$

2. $f(s) = \frac{3s(s^2+5)(s^2+8)}{(s^2+1)(s^2+6)}$, $f(j\omega) = j\frac{3\omega(5-\omega^2)(8-\omega^2)}{(1-\omega^2)(6-\omega^2)}$

$\Rightarrow Re[f(j\omega)] = 0 \Rightarrow f(s) + f(-s) = 0$ on $s = j\omega$

$\Rightarrow$ lossless positive-real

2nd Foster

$y(s) = \frac{3s(s^2+5)(s^2+8)}{(s^2+1)(s^2+6)} = k_\infty s + \frac{k_0}{s} + \sum \frac{2k_i s}{s^2+\omega_i^2}$

$= k_\infty s + \frac{2k_2 s}{s^2+1} + \frac{2k_4 s}{s^2+6} = 3s + \frac{2k_2 s}{s^2+1} + \frac{2k_4 s}{s^2+6}$

$k_\infty = 3$

$\left(\frac{s^2+1}{s}\right) y(s) = \frac{3(s^2+5)(s^2+8)}{s^2+6} = (s^2+1)k_\infty + 2k_2 + \frac{2k_4}{s^2+6}(s^2+1)$

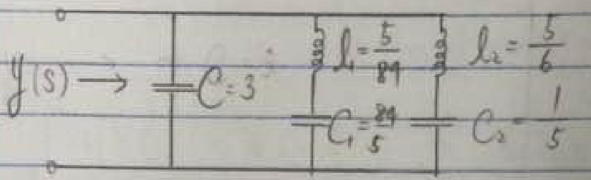
$s^2 = -1 \Rightarrow \frac{3\times 4\times 7}{5} = 2k_2$, $k_2 = \frac{42}{5}$

$\left(\frac{s^2+6}{s}\right) y(s)\Big|_{s^2=-6} = \frac{3(s^2+5)(s^2+8)}{s^2+1}\Big|_{s^2=-6} = \frac{-3\times 2}{-5} = (s^2+6)k_\infty + \frac{2k_2(s^2+6)}{s^2+1} + 2k_4\Big|_{s^2=-6}$
$= 0 + 0 + 2k_4$

$\Rightarrow k_4 = \frac{3}{5}$

$$y(S) = 3S + \frac{\frac{84}{5}s}{s^2+1} + \frac{\frac{6}{5}s}{s^2+6}$$

$$z_1 = \frac{1}{y_1} = \frac{s^2+1}{\frac{84}{5}s} = \frac{5}{84}s + \frac{1}{\frac{84}{5}s}, \quad z_2 = \frac{1}{y_2} = \frac{s^2+6}{\frac{6}{5}s} = \frac{5}{6}s + \frac{1}{\frac{1}{5}s}$$



2nd Foster

$$z(s) = \frac{1}{y(s)} = \frac{(s^2+1)(s^2+6)}{3s(s^2+5)(s^2+8)} = \frac{k_0}{s} + \frac{2k_2 s}{s^2+5} + \frac{2k_4 s}{s^2+8}$$

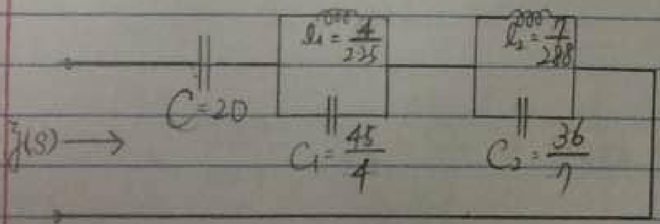
$$\frac{s^2+5}{s}\cdot z(s)\Big|_{s^2=-5} = \frac{(s^2+1)(s^2+6)}{3s^2(s^2+8)}\Big|_{s^2=-5} = \frac{-4\times 1}{3\times(-5)\times 3} = 0 + 2k_2 + 0 \Rightarrow k_2 = \frac{2}{45}$$

$$\frac{s^2+8}{s}\cdot z(s)\Big|_{s^2=-8} = \frac{(-7)(-2)}{3\times(-8)\times(-3)} = 0 + 0 + 2k_4 \Rightarrow k_4 = \frac{7}{72}$$

$$3\left[k_0(s^2+5)(s^2+8) + \frac{4}{45}(s^4+8s^2) + \frac{14}{72}(s^4+5s^2)\right] = (s^2+1)(s^2+6)$$

$$120k_0 = 6 \Rightarrow k_0 = \frac{1}{20}$$

$$z(s) = \frac{\frac{1}{20}}{s} + \frac{\frac{4}{45}s}{s^2+5} + \frac{\frac{14}{72}s}{s^2+8} = \frac{1}{20s} + \frac{1}{\frac{45}{4}s + \frac{1}{\frac{4}{225}s}} + \frac{1}{\frac{36}{7}s + \frac{1}{\frac{7}{288}s}}$$



1st Foster

1st Foster: components are first connected in parallel then in series, using the input impedance to construct it.

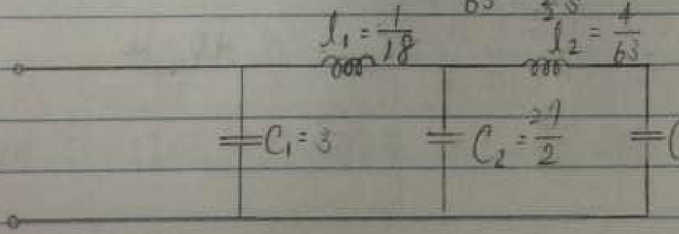
2nd Foster components are first connected in series then in parallel, using the input admittance to construct it.

1st Cauer

$$Y(s) = \frac{3s^5+39s^3+120s}{s^4+7s^2+6}$$ remove poles at ∞

$$\begin{array}{l}
s^4+7s^2+6 \;\big)\; 3s^5 + 39s^3 + 120s \quad \rightarrow 3s \\
\qquad\qquad 3s^5 + 21s^3 \\
\qquad\qquad\qquad 18s^3 + 102s \;\big)\; s^4+7s^2+6 \quad \rightarrow \tfrac{1}{18}s \\
\qquad\qquad\qquad\qquad s^4 + \tfrac{17}{3}s^2 \\
\qquad\qquad\qquad\qquad\qquad \tfrac{4}{3}s^2+6 \;\big)\; 18s^3+102s \quad \rightarrow \tfrac{27}{2}s \\
\qquad\qquad\qquad\qquad\qquad\qquad 18s^3+81s \\
\qquad\qquad\qquad\qquad\qquad\qquad\qquad 21s \;\big)\; \tfrac{4}{3}s^2+6 \quad \rightarrow \tfrac{4}{63}s \\
\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \tfrac{4}{3}s^2 \\
\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad 6 \;\big)\; 21s \quad \rightarrow \tfrac{7}{2}s \\
\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad 21s \\
\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad 0
\end{array}$$

$$\Rightarrow Y(s) = 3s + \cfrac{1}{\frac{1}{18}s + \cfrac{1}{\frac{27}{2}s + \cfrac{1}{\frac{4}{63}s + \cfrac{1}{\frac{7}{2}s}}}}, \quad Z(s) = 0s + \cfrac{1}{3s + \cfrac{1}{\frac{1}{18}s + \cfrac{1}{\frac{27}{2}s + \cfrac{1}{\frac{4}{63}s + \cfrac{1}{\frac{7}{2}s}}}}}$$



1st Cauer

✓

2nd Cauer remove poles at 0

$$Z(s) = \frac{6+7s^2+s^4}{120s+39s^3+3s^5}$$

$$\frac{1}{20s}$$

$$120s+39s^3+3s^5 \,\big)\, 6+7s^2+s^4$$

$$6+\frac{39}{20}s^2+\frac{3}{20}s^4 \qquad \frac{2400}{101s}$$

$$\frac{101}{20}s^2+\frac{17}{20}s^4 \,\big)\, 120s+39s^3+3s^5$$

$$120s+\frac{17430}{101}s^3 \qquad \frac{10201}{37980s}$$

$$\frac{1899}{101}s^3+3s^5 \,\big)\, \frac{101}{20}s^2+\frac{17}{20}s^4$$

$$\frac{101}{20}s^2+\frac{101\times101}{633\times20}s^4 \qquad \frac{1202067}{2828s}$$

$$\frac{28}{633}s^4 \,\big)\, \frac{1899}{101}s^3+3s^5$$

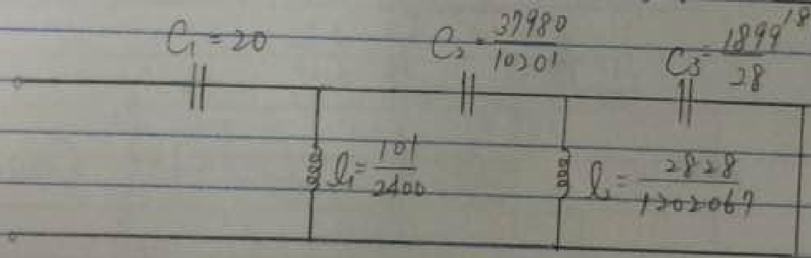
$$\frac{1899}{101}s^3 \qquad \frac{28}{18993}$$

$$3s^5 \,\big)\, \frac{28}{633}s^4$$

$$\frac{28}{633}s^4$$

$$0$$

$$\Rightarrow Z(s) = \frac{1}{20s} + \cfrac{1}{\frac{2400}{101s} + \cfrac{1}{\frac{10201}{37980s} + \cfrac{1}{\frac{1202067}{2828s} + \cfrac{1}{\frac{28}{18993}}}}}$$



✓ 2nd Cauer

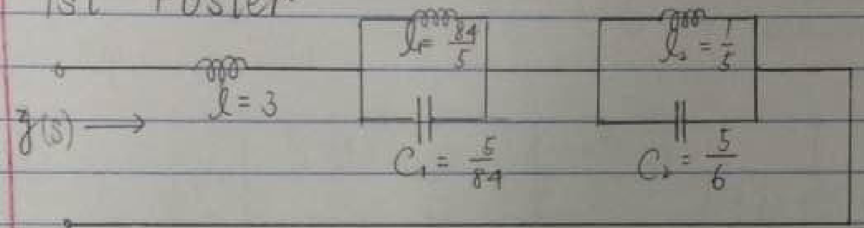
1st Cauer: the L's are horizontal while the C's are vertical. We choose the reaction function (could be $y(s)$ or $z(s)$) that has the numerator with larger degree

2nd Cauer: the C's are horizontal while the L's are vertical. We choose the reaction function that has the numerator with the smallest degree (e.g. constant)

$f(s)$ is an impedance $\Rightarrow$ dual circuit

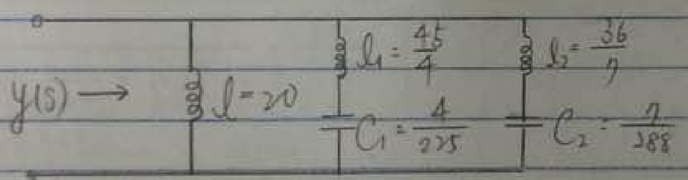
$$z(s) = \frac{3s(s^2+5)(s^2+8)}{(s^2+1)(s^2+6)} = 3s + \frac{1}{\frac{5}{84}s + \frac{1}{\frac{84}{5}s}} + \frac{1}{\frac{5}{6}s + \frac{1}{\frac{1}{5}s}}$$

1st Foster



$$y(s) = \frac{(s^2+1)(s^2+6)}{3s(s^2+5)(s^2+8)} = \frac{1}{20s} + \frac{1}{\frac{45}{4}s + \frac{1}{\frac{4}{225}s}} + \frac{1}{\frac{36}{7}s + \frac{1}{\frac{7}{288}s}}$$

2nd Foster



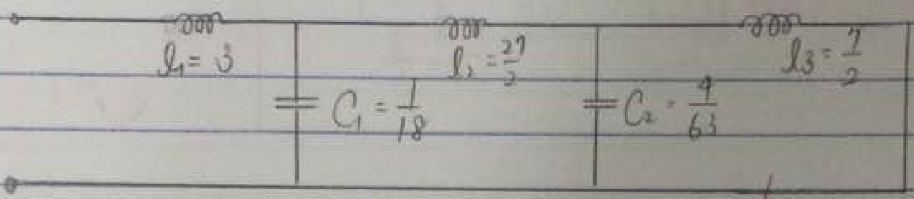
1st Foster of $f(s)$ as an impedance is the dual of 2nd Foster of $f(s)$ as an admittance.

2nd Foster of $f(s)$ as a $z(s)$ is the dual of 1st Foster of $f(s)$ as a $y(s)$

As for the Cauer with $f(s)$ as an impedance

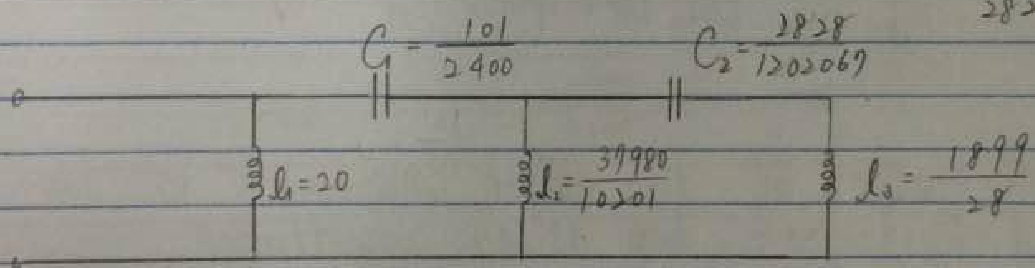
1st Cauer

$$y(s) = \frac{s^4+7s^2+6}{3s^5+39s^3+120s}, \quad z(s) = \frac{3s^5+39s^3+120s}{s^4+7s^2+6} = 3s + \cfrac{1}{\frac{1}{18}s + \cfrac{1}{\frac{27}{2}s + \cfrac{1}{\frac{4}{63}s + \cfrac{1}{\frac{7}{2}s}}}}$$



2nd Cauer

$$y(s) = \frac{6+7s^2+s^4}{3s^5+39s^3+120s} = \frac{1}{20s} + \cfrac{1}{\frac{2400}{101s} + \cfrac{1}{\frac{10201}{37980s} + \cfrac{1}{\frac{1202067}{2828s} + \cfrac{1}{\frac{28}{1899s}}}}}$$



1st Cauer with $f(s)$ as a $z(s)$ is the dual of
1st Cauer with $f(s)$ as a $y(s)$

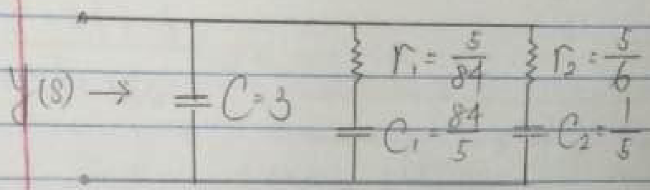
2nd Cauer with $f(s)$ as a $z(s)$ is the dual of
2nd Cauer with $f(s)$ as a $y(s)$

3 admittance synthesis $y_{LC}(s) = \frac{3s(s^2+5)(s^2+8)}{(s^2+1)(s^2+6)}$

$$= 3s + \frac{\frac{84}{5}s}{s^2+1} + \frac{\frac{6}{5}s}{s^2+6}$$

LC → RC

2nd Foster:



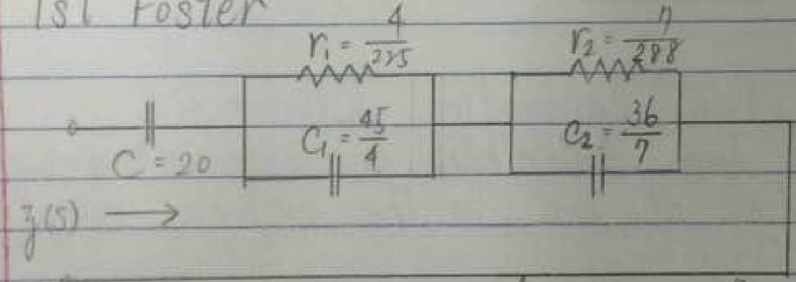
$$y_{RC}(s) = k_\infty s + k_0 + \sum \frac{k_i s}{s+w_i^2}$$

$$= 3s + \frac{\frac{84}{5}s}{s+1} + \frac{\frac{6}{5}s}{s+6}$$

$$= 3s + \frac{1}{\frac{5}{84}+\frac{1}{\frac{84}{5}s}} + \frac{1}{\frac{5}{6}+\frac{1}{\frac{1}{5}s}}$$

$$= \frac{3s(s+5)(s+8)}{(s+1)(s+6)}$$

1st Foster

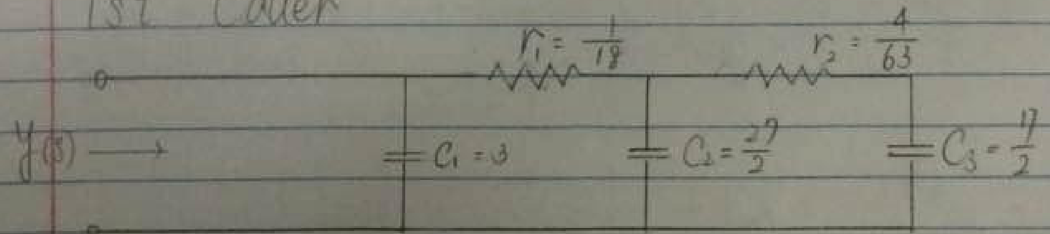


$$z_{RC}(s) = \frac{1}{20s} + \frac{\frac{4}{45}}{s+5} + \frac{\frac{7}{36}}{s+8} = \frac{\frac{1}{20}}{s} + \frac{1}{\frac{45}{4}s+\frac{1}{\frac{4}{225}}} + \frac{1}{\frac{36}{7}s+\frac{1}{\frac{7}{288}}}$$

$$= \frac{(s+1)(s+6)}{3s(s+5)(s+8)}$$

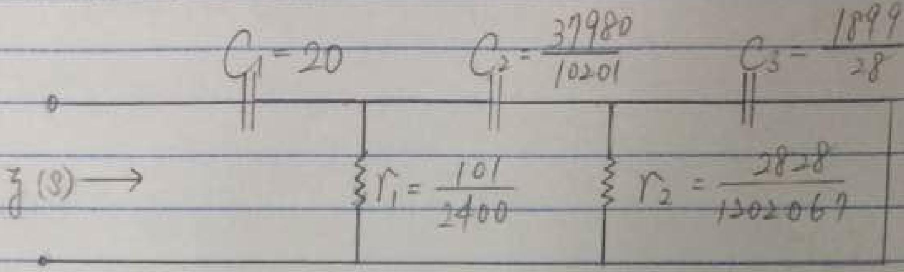
$$\Rightarrow y_{RC}(s) = \frac{3s(s+5)(s+8)}{(s+1)(s+6)}$$

1st Cauer



$$y_{RC}(s) = 3s + \frac{1}{\frac{1}{18}+\frac{1}{\frac{27}{2}s+\frac{1}{\frac{4}{63}+\frac{1}{\frac{17}{2}s}}}} = \frac{3s^3+39s^2+120s}{s^2+7s+6}$$

2nd Cauer



$$Z_{RC}(s) = \frac{1}{20s} + \cfrac{1}{\frac{2400}{101} + \cfrac{1}{\frac{10201}{37980s} + \cfrac{1}{\frac{1202067}{2828} + \cfrac{1}{\frac{28}{1899s}}}}} = \frac{6+7s+s^2}{120s+39s^2+3s^3}$$

$$\Rightarrow Y_{RC}(s) = \frac{3s^3+39s^2+120s}{s^2+7s+6}$$

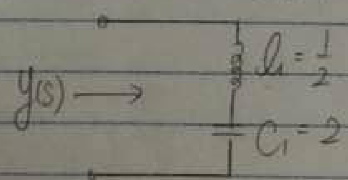
Note that when converting $LC \rightarrow RC$ we take off the variable s associated with the numerical values of L's in the original equation

4.

$$S(s) = \frac{3s^2-6s+3}{3s^2+6s+3} = \frac{s^2-2s+1}{s^2+2s+1}$$

$$y(s) = (1+S)^{-1}(1-S) = \frac{s^2+2s+1}{2s^2+2} \times \frac{4s}{s^2+2s+1} = \frac{2s}{s^2+1}$$

$$= \cfrac{1}{\frac{s}{2} + \frac{1}{2s}}$$

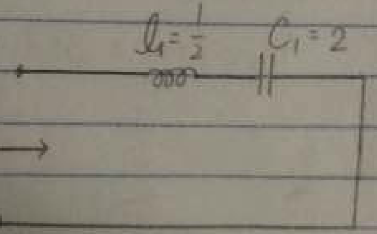


2nd Foster

$$Z(s) = \frac{s^2+1}{2s} = k_\infty s + \frac{k_0}{s} \qquad s \cdot Z(s)\Big|_{s=0} = \frac{1}{2} = 0 + k_0, \quad k_0 = \frac{1}{2}$$

$$\frac{s^2+1}{2s} = k_\infty s + \frac{1}{2s}$$

$$\Rightarrow k_\infty = \frac{1}{2}, \quad Z(s) = \frac{1}{2}s + \frac{1}{2s}$$



1st Foster

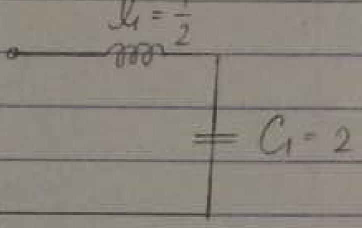
In this case, the 1st Foster and the 2nd Foster are the same circuit

1st Cauer

$$Z(s) = \frac{s^2+1}{2s}$$

$$= \frac{1}{2}s + \frac{1}{2s+0}$$

$$\begin{array}{r|l} & \frac{1}{2}s \\ 2s & s^2+1 \\ & s^2 \\ \hline & \quad 1 \end{array} \qquad \begin{array}{r|l} & 2s \\ 1 & 2s \\ & 2s \\ \hline & 0 \end{array}$$

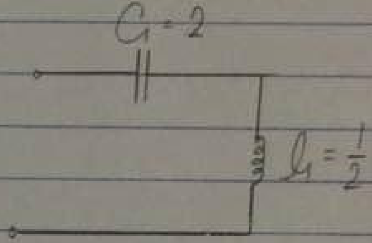


2nd Cauer:

$$Z(s) = \frac{1+s^2}{2s}$$

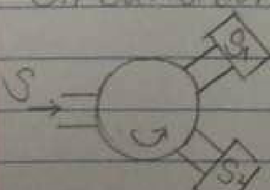
$$= \frac{1}{2s} + \frac{1}{\frac{2}{s}+0}$$

$$\begin{array}{r|l} & \frac{1}{2s} \\ 2s & 1+s^2 \\ & 1 \\ \hline & s^2 \end{array} \qquad \begin{array}{r|l} & \frac{2}{s} \\ s^2 & 2s \\ & 2s \\ \hline & 0 \end{array}$$



1st Cauer and 2nd Cauer are also the same

Circulator



we know from the notes $S = S_1 \cdot S_2$

$$S = \frac{s^2-2s+1}{s^2+2s+1} = \frac{(s-1)^2}{(s+1)^2} = S_1 \cdot S_2 = \frac{s-1}{s+1} \cdot \frac{s-1}{s+1}$$

$$y_1 = y_2 = (1+S_1)^{-1}(1-S_1) = \frac{s+1}{2s} \cdot \frac{2}{s+1} = \frac{1}{s} \Rightarrow L = 1$$