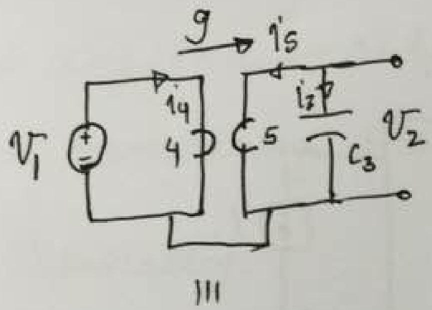
ASSIGNMENT-3

SIDDHARTH TYAGI ENEE-610 : NETWORK THEORY.

1

(a) Given circuit:



Using notation used in class:-
we can obtain the Y_{b-2} of the two port network with V_1 ~~as input~~ and V_2 as the two ports:-

We know:

$$V_3 = \left(\frac{1}{sC_3}\right) i_3 \Rightarrow i_3 = (sC_3) \cdot V_3$$

$$\begin{bmatrix} i_4 \\ i_5 \end{bmatrix} = \begin{bmatrix} 0 & -g \\ g & 0 \end{bmatrix} \begin{bmatrix} V_4 \\ V_5 \end{bmatrix}$$

This yields the Y_{b-2} of the two port network as:-

$$Y_{b-2} = \begin{bmatrix} sC_3 & 0 & 0 \\ 0 & 0 & -g \\ 0 & g & 0 \end{bmatrix}$$

Now; we saw, the Y^a_{b-2} of the adjoint needs to be such that:-

$$\boxed{Y^a_{b-2} = Y^T_{b-2} = \begin{bmatrix} sC_3 & 0 & 0 \\ 0 & 0 & g \\ 0 & -g & 0 \end{bmatrix}}$$

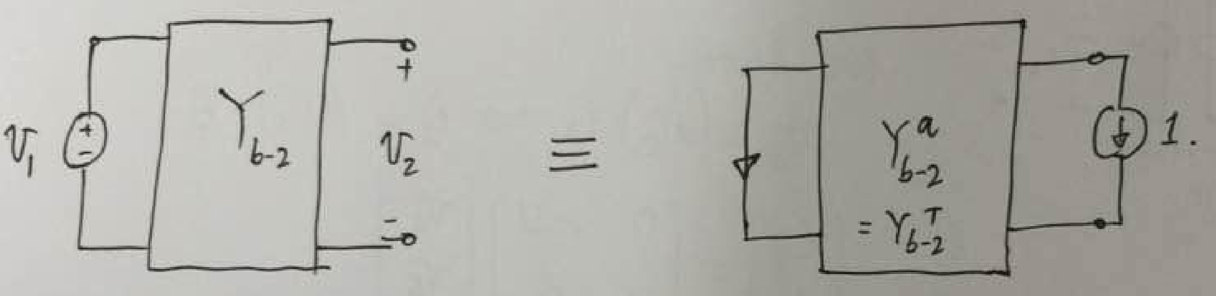
clearly, branch 3 will remain capacitor

branch 4/5 : a gyrator with inverted direction.

Thus: to obtain the adjoint circuit: the 2-port network is given by Y^{a}_{b-2}.

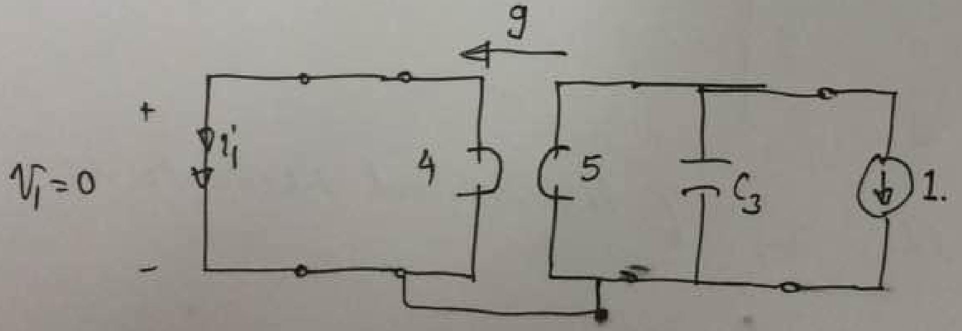
Also; a voltage-source (input v_1) becomes a <u>short-circuit</u> in adjoint, and the open-port (v_2) becomes a <u>current-source of value 1.</u>

i.e.



original ckt. adjoint

∴ The adjoint can be drawn as:-



(b) To obtain the sensitivity using adjoint:-

Since both adjoint and original circuit have the same graph:

$$\boxed{V_b^T i_b^a - (V_b^a)^T i_b = 0}$$

Let input-and-output ports (v_1 and v_2) be $\underline{V_{b1} \text{ and } V_{b2}}$

and remaining branches be $\underline{V_{b-2}}$.

∴ Separating:

$$\left(V_{b1} i_{b1}^a + V_{b2} i_{b2}^a + V_{b-2}^T i_{b-2}^a\right) - \left(V_{b1}^a i_{b1} + V_{b2}^a i_{b2} + (V_{b-2}^a)^T i_{b-2}\right) = 0$$

If 'x' be a parameter; taking ~~the~~ derivative of above eqn wrt x' and noting that derivatives of all adjoint terms = 0

we get:-

$$\left\{\frac{dv_{b1}}{dx} i_{b1}^a + \frac{dv_{b2}}{dx} i_{b2}^a + \left(\frac{d}{dx} V_{b-2}^T\right) i_{b-2}^a\right\} - \left\{V_{b1}^a \frac{di_{b1}}{dx} + V_{b2}^a \frac{di_{b2}}{dx} + (V_{b-2}^a)^T \frac{di_{b-2}}{dx}\right\} = 0$$

also $i_{b2} = 0$ [∵ open]

if $V_{b1} = 1$ (constant) $\Rightarrow T = \frac{V_{b2}}{V_{b1}} = V_{b2}$.

and if we choose $i_{b2}^a = 1$, above simplifies to:

$$\therefore \frac{dv_{b2}}{dx} = V_{b1}^a \frac{di_{b1}}{dx} + \left[(V_{b-2}^a)^T \frac{d\, i_{b-2}}{dt} - \frac{d}{dx}(V_{b-2})^T \cdot i_{b-2}^a\right]$$

using: $i_{b-2} = Y_{b-2} v_{b-2}$; $i^a_{b-2} = Y^a_{b-2} v^a_{b-2}$; above can be expressed as:-

$$\frac{dv_{b2}}{dx} = v^a_{b1} \frac{di_{b1}}{dx} + \frac{dv^T_{b-2}}{dx}\left[Y^a_{b-2} - Y^T_{b-2}\right] \cdot v^a_{b-2} + \left[(v^a_{b-2})^T \frac{dY}{dx} v_{b-2}\right]$$

If $v^a_{b1} = 0$; and $Y^a_{b-2} = Y^T_{b-2}$

$$\Rightarrow \boxed{\frac{dT}{dx} = +(v^a_{b-2})^T \frac{dY_{b-2}}{dx} v_{b-2}} \qquad (\because v_{b2} = T \text{ when } v_{b1} = 1).$$

(c) The given $T = \frac{v_2}{v_1}$;

When $\underline{\underline{x = C_3}}$:- $\quad v^a_{b-2} = \begin{bmatrix} v^a_3 \\ v^a_4 \\ v^a_5 \end{bmatrix} \quad v_{b-2} = \begin{bmatrix} v_3 \\ v_4 \\ v_5 \end{bmatrix}$

$$\therefore \frac{dT}{dC_3} = -\left[v^a_3\ v^a_4\ v^a_5\right] . \frac{d}{dC_3}\begin{bmatrix} sC_3 & 0 & 0 \\ 0 & 0 & -g \\ 0 & g & 0 \end{bmatrix} . \begin{bmatrix} v_3 \\ v_4 \\ v_5 \end{bmatrix}$$

$$= -\left[v^a_3\ v^a_4\ v^a_5\right] \begin{bmatrix} s & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} v_3 \\ v_4 \\ v_5 \end{bmatrix}$$

$$= -s . v^a_3 v_3 =$$

$$v_3 = -g \cdot \left(\frac{1}{sC_3}\right) = -\frac{g}{sC_3}$$

$$v^a_3 = -\frac{1}{sC_3}$$

$$\therefore \boxed{\frac{dT}{dC_3} = +\frac{g}{sC_3^2}}$$

$$\therefore S^T_{C_3} = \left(\frac{\frac{dT}{dC_3}}{\frac{T}{C_3}}\right) = \frac{-\frac{g}{sC_3^2}}{-\frac{g}{sC_3^2}} \qquad \left(\because T = -\frac{g}{sC_3}\right)$$

$$\boxed{S^T_{C_3} = -1}$$

When $\underline{\underline{x = g}}$

$$\therefore \frac{dT}{dg} = +\begin{bmatrix} v_3^a & v_4^a & v_5^a \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} v_3 \\ v_4 \\ v_5 \end{bmatrix}$$

$$= +\begin{bmatrix} v_3^a & v_4^a & v_5^a \end{bmatrix} \begin{bmatrix} 0 \\ -v_5 \\ v_4 \end{bmatrix}$$

$$\therefore \frac{dT}{dg} = +\left\{ -v_4^a v_5 + v_5^a v_4 \right\}$$

$v_4^a = 0$

$v_5^a = -\frac{1}{sC_3}$

$v_4 = 1.$

$T = -\frac{g}{sC_3}$

$$\therefore \boxed{\frac{dT}{dg} = \frac{-1}{sC_3}}$$

$$\therefore \boxed{S^T_g = \frac{\frac{dT}{dg}}{\frac{T}{g}} = \frac{-\frac{1}{sC_3}}{\frac{-\frac{g}{sC_3}}{g}} = 1.}$$

(d)

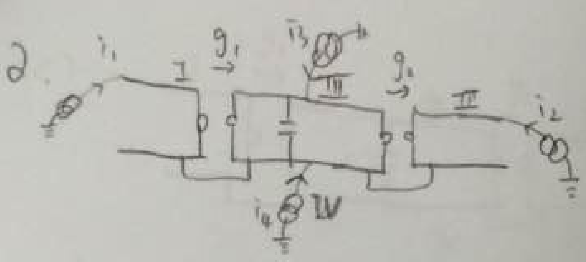
$$T = \frac{-g}{sC_3} \Rightarrow \boxed{\frac{dT}{dC_3} = \frac{g}{sC_3^2}} \quad \text{(as earlier)}$$

$$\therefore \; S^{T}_{C_3} = \frac{\frac{g}{sC_3^2}}{\frac{-g}{sC_3^2}} = -1.$$

also; $\boxed{\frac{dT}{dg} = \frac{-1}{sC_3^2}}$ (as earlier)

Anqi Deng

2.



a). $i_1 = g_1(v_3 - v_4)$

$i_2 = -g_2(v_3 - v_4)$

$i_3 = -g_1(v_1 - v_4) + sC(v_3 - v_4) + g_2(v_2 - v_4)$

$i_4 = -(i_1 + i_2 + i_3)$

$$\begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 0 & g_1 & -g_1 \\ 0 & 0 & -g_2 & g_2 \\ -g_1 & g_2 & sC & g_1 - g_2 - sC \\ g_1 & -g_2 & g_2 - g_1 - sC & sC \end{bmatrix}}_{Y_{4\times4}} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix}$$

b). Ground node IV. $i_4 = -(i_1 + i_2 + i_3)$

$v_4 = 0$

$$\begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & g_1 \\ 0 & 0 & -g_2 \\ -g_1 & g_2 & sC \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$$

eliminate III $i_3 = 0$

$$i_3 = [-g_1 \quad g_2] \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} + [sC][v_3] = 0$$

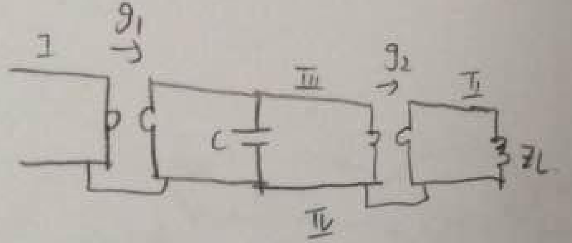
$$v_3 = -\frac{1}{sC}[-g_1 \quad g_2] \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} + \begin{bmatrix} g_1 \\ -g_2 \end{bmatrix} \cdot \left(-\frac{1}{sC}\right) \cdot [-g_1 \quad g_2] \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

$$Y_{2port} = \left(\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} - \frac{1}{sC} \begin{bmatrix} -g_1^2 & g_1 g_2 \\ g_1 g_2 & -g_2^2 \end{bmatrix} \right) \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

$$= \frac{1}{sC} \begin{bmatrix} g_1^2 & -g_1 g_2 \\ -g_1 g_2 & g_2^2 \end{bmatrix}$$ since it is singular, no Z_{ij} exist

c)



$$i_1 = g_1 (V_3 - V_4)$$

$$i_2 = -g_2 (V_3 - V_4) + \frac{1}{ZL}(V_2 - V_4)$$

$$i_3 = -g_1 (V_1 - V_4) + sC (V_3 - V_4) + g_2 (V_2 - V_4)$$

$$i_4 = -(i_1 + i_2 + i_3)$$

$$Y_{4\times4} = \begin{bmatrix} 0 & 0 & g_1 & -g_1 \\ 0 & \frac{1}{ZL} & -g_2 & g_2 - \frac{1}{ZL} \\ -g_1 & g_2 & sC & g_1 - g_2 - sC \\ g_1 & g_2 - \frac{1}{ZL} & g_2 - g_1 - sC & \frac{1}{ZL} + sC \end{bmatrix}$$

since $i_4 = -(i_1 + i_2 + i_3)$

$$\text{so } Y_{3\times3} = \begin{bmatrix} 0 & 0 & g_1 \\ 0 & \frac{1}{ZL} & -g_2 \\ -g_1 & g_2 & sC \end{bmatrix}$$

eliminate III $i_3 = 0$

$$i_3 = \begin{bmatrix} -g_1 & g_2 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} + sC[V_3] = 0$$

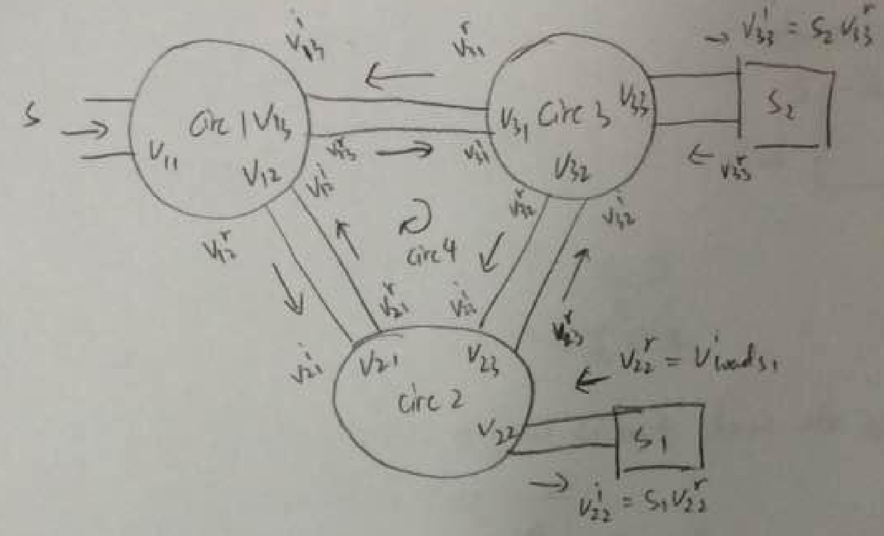
$$V_3 = -\frac{1}{sC} \begin{bmatrix} -g_1 & g_2 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$Y_{2port} = \begin{bmatrix} 0 & 0 \\ 0 & \frac{1}{ZL} \end{bmatrix} - \frac{1}{sC} \begin{bmatrix} -g_1^2 & g_1 g_2 \\ g_1 g_2 & -g_2^2 \end{bmatrix} = \begin{bmatrix} \frac{g_1^2}{sC} & -\frac{g_1 g_2}{sC} \\ -\frac{g_1 g_2}{sC} & \frac{1}{ZL} + \frac{g_2^2}{sC} \end{bmatrix}$$

$$Z = Y^{-1}$$

input impedence $Z_{in} = \frac{1}{\Delta} \cdot \left(\frac{1}{ZL} + \frac{g_2^2}{sC}\right) = \frac{1}{\frac{g_1^2}{sC}\left(\frac{1}{ZL} + \frac{g_2^2}{sC}\right) - \left(\frac{g_1 g_2}{sC}\right)^2} \cdot \left(\frac{1}{ZL} + \frac{g_2^2}{sC}\right) = \frac{sC + ZL \cdot g_2^2}{g_1^2}$

3



since all loaded are normalized to $Z_0=1$. all terms can be depended only on incident V^i and reflected V^r

counted counter clockwise

$V_{12}^r = V_{11}^i$ $V_{22}^r = V_{21}^i$ $V_{32}^r = V_{31}^i$, $V_{11}^r = V_{31}^i$, $V_{12}^r = V_{32}^i$ $V_{13}^r = V_{33}^i$

and $V_{21}^i = V_{12}^r$ $V_{13}^i = V_{31}^r$ $V_{32}^i = V_{23}^r$

$$V_{11}^r = V_{13}^i = V_{31}^r = V_{33}^i = S_2 V_{33}^r = S_2 V_{32}^i = S_2 V_{23}^r = S_2 V_{22}^i = S_1 S_2 V_{22}^r$$

$$= S_1 S_2 V_{21}^i = S_1 S_2 V_{12}^r = S_1 S_2 V_{11}^i$$

so $S = S_1 S_2$

So it is lossless.

Anomaly there exist a circulator 4 which towards clockwise among the inner side of these three circulator, which will cause reduction of the efficiency