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ENEE 610 HW2

1.
(a) $\frac{dx_1}{dt} = 3x_1 - 5x_2 + 9u$

$\frac{dx_2}{dt} = 2\tanh(3x_1 - 3x_2) + 12x_2$

$y = 2x_1 - 3x_2 + 8u$

$\dot{x} = Ax + Bu$
$y = Cx + Du$

$x_1 = x_2 = 0$

Taylor series:

$$\begin{bmatrix} \frac{d(x_1-0)}{dt} \\ \frac{d(x_2-0)}{dt} \end{bmatrix} = \begin{bmatrix} \frac{\partial f_1}{\partial x_1}\Big|_{x_1=0,\,x_2=0} & \frac{\partial f_1}{\partial x_2}\Big|_{x_1=0,\,x_2=0} \\ \frac{\partial f_2}{\partial x_1}\Big|_{x_1=0,\,x_2=0} & \frac{\partial f_2}{\partial x_2}\Big|_{x_1=0,\,x_2=0} \end{bmatrix} \begin{bmatrix} x_1-0 \\ x_2-0 \end{bmatrix} + \begin{bmatrix} \frac{\partial f_1}{\partial u}\Big|_{x_1=0,\,x_2=0} \\ \frac{\partial f_2}{\partial u}\Big|_{x_1=0,\,x_2=0} \end{bmatrix} u$$

$$\begin{bmatrix} \frac{dx_1}{dt} \\ \frac{dx_2}{dt} \end{bmatrix} = \begin{bmatrix} 3 & -5 \\ 6 & 6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 9 \\ 0 \end{bmatrix} u$$

$$y = \begin{bmatrix} 2 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + 8u$$

✓

(b) Laplace Transform of the state equation

$$sX = AX + BU$$
$$Y = CX + DU$$

$$\Rightarrow (sI_n - A)X = BU \quad , \quad X = (sI_n - A)^{-1}BU$$

$$Y = C(sI_n - A)^{-1}BU + DU = (C(sI_n - A)^{-1}B + D)U$$

Transfer function $= C(sI_n - A)^{-1}B + D$

$$A = \begin{bmatrix} 3 & -5 \\ 6 & 6 \end{bmatrix} \quad B = \begin{bmatrix} g \\ 0 \end{bmatrix} \quad C = \begin{bmatrix} 2 & -3 \end{bmatrix} \quad D = 8$$

$$T(s) = \frac{1}{(s-3)(s-6)+30}\left\{ \begin{bmatrix} 2 & -3 \end{bmatrix} \begin{bmatrix} s-6 & -5 \\ 6 & s-3 \end{bmatrix} \begin{bmatrix} g \\ 0 \end{bmatrix} \right\} + 8$$

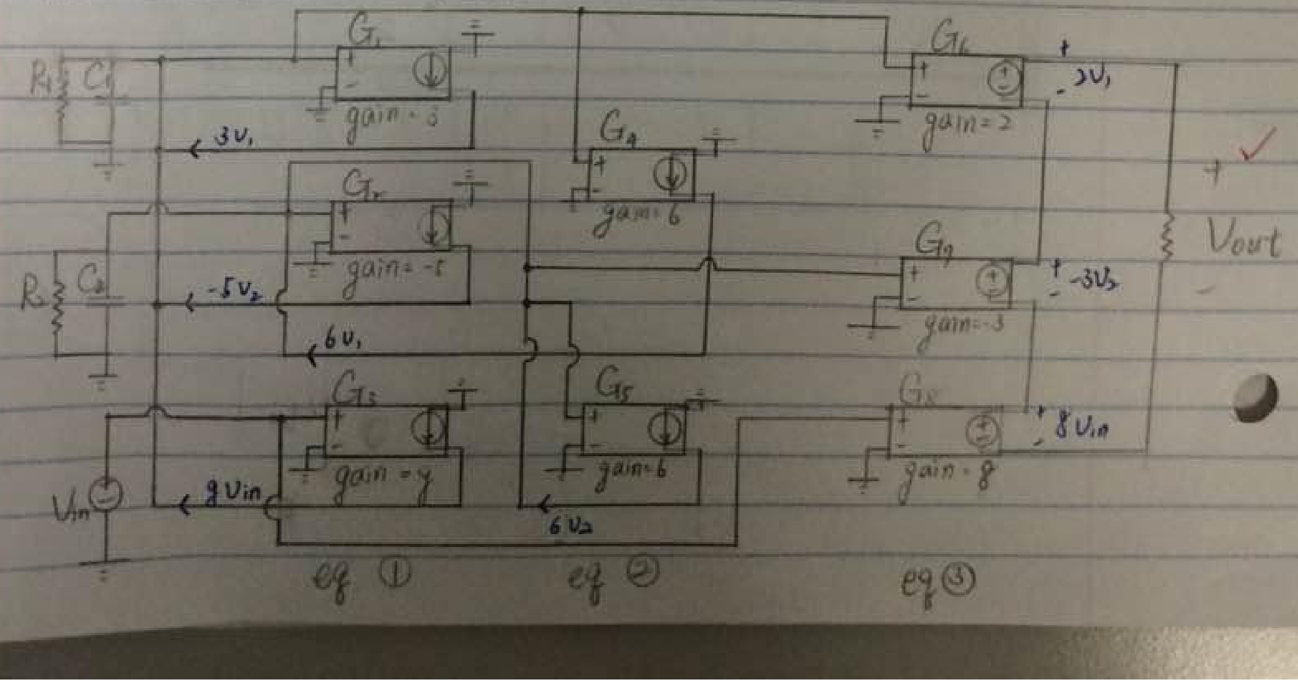
$$= \frac{(2s-30)g + 8(s^2-9s+48)}{s^2-9s+48} = \frac{8s^2+(2g-72)s+(384-30g)}{s^2-9s+48}$$

(c) For the linearized system after expanded as Taylor series

$dx_1/dt = 3x_1 - 5x_2 + g\,v_{in}$ $\qquad x_1 = v_1$

$dx_2/dt = 6x_1 + 6x_2$ $\qquad x_2 = v_2$

$V_{out} = 2x_1 - 3x_2 + 8V_{in}$



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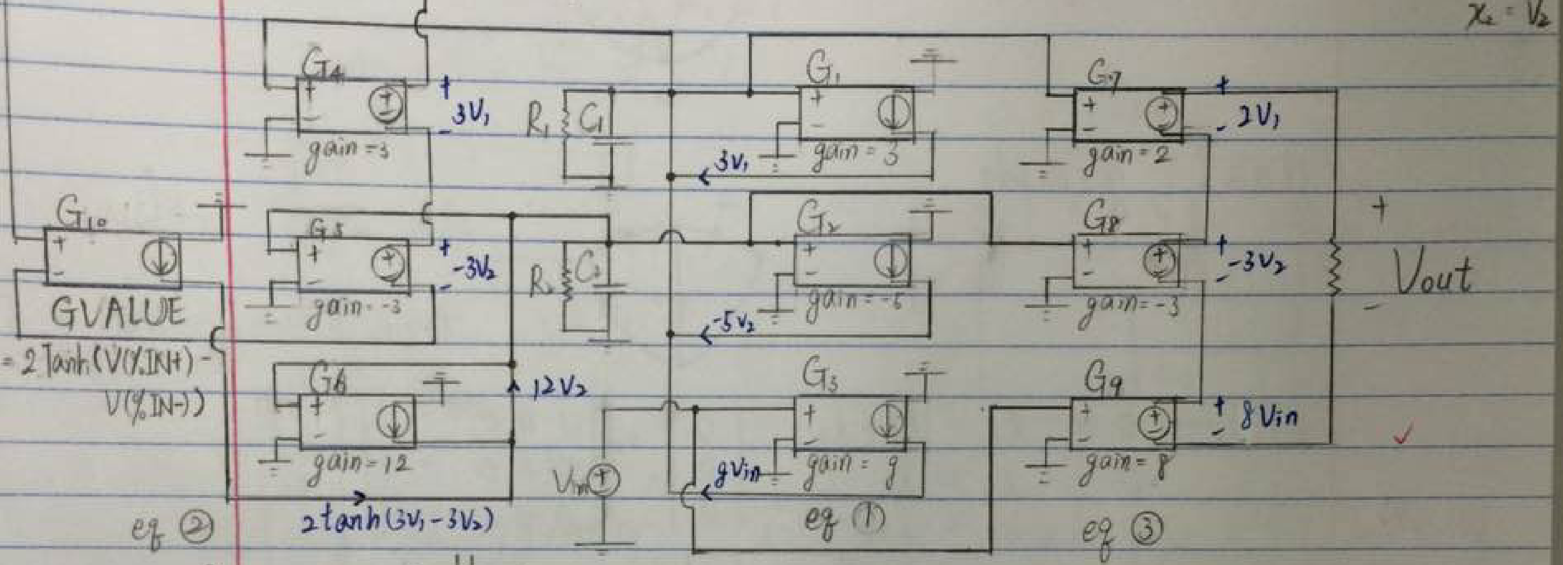
(d) nonlinear eq. ② ⇒ $dx_2/dt = 2\,\mathrm{Tanh}(3x_1 - 3x_2) + 12x_2$, $x_1 = V_1$, $x_2 = V_2$



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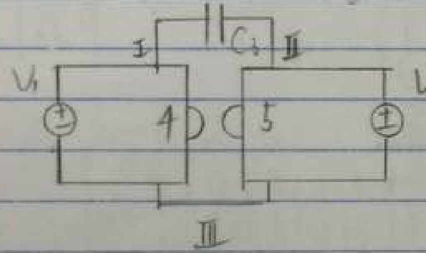
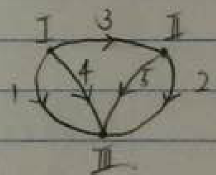
(a)



directed graph



tree branches: 1, 2

KCL $0 = i_1 + i_3 + i_4$
$0 = i_2 - i_3 + i_5$

$$\begin{bmatrix} 0 \\ 0 \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & -1 & 0 & 1 \end{bmatrix}}_{K} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \end{bmatrix}$$

cut set matrix $e = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & -1 & 0 & 1 \end{bmatrix}$ ✓

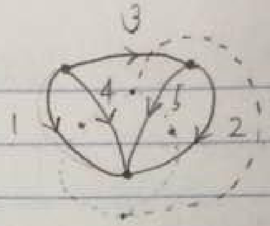
KVL: $0 = V_3 - V_1 + V_2$
$0 = V_4 - V_1$
$0 = V_5 - V_2$

$$\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -1 & 1 & 1 & 0 & 0 \\ -1 & 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \end{bmatrix}$$

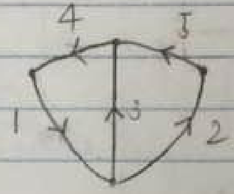
$-K^T$

tie set matrix $J = \begin{bmatrix} -1 & 1 & 1 & 0 & 0 \\ -1 & 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 & 1 \end{bmatrix}$ ✓

(b) original graph



dual graph



tree branches. 3, 4, 5

KCL: $0 = i_3 - i_1 + i_2$
$0 = i_4 - i_1$
$0 = i_5 - i_2$

$$\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = \left[\begin{array}{ccc|cc} 1 & 0 & 0 & -1 & 1 \\ 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & -1 \end{array}\right] \begin{bmatrix} i_3 \\ i_4 \\ i_5 \\ i_1 \\ i_2 \end{bmatrix}$$

cut set matrix $e' = \left[\begin{array}{ccc|cc} 1 & 0 & 0 & -1 & 1 \\ 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & -1 \end{array}\right]$

KVL. $0 = V_1 + V_3 + V_4$
$0 = V_2 - V_3 + V_5$

$$\begin{bmatrix} 0 \\ 0 \end{bmatrix} = \left[\begin{array}{ccc|cc} 1 & 1 & 0 & 1 & 0 \\ -1 & 0 & 1 & 0 & 1 \end{array}\right] \begin{bmatrix} V_3 \\ V_4 \\ V_5 \\ V_1 \\ V_2 \end{bmatrix}$$

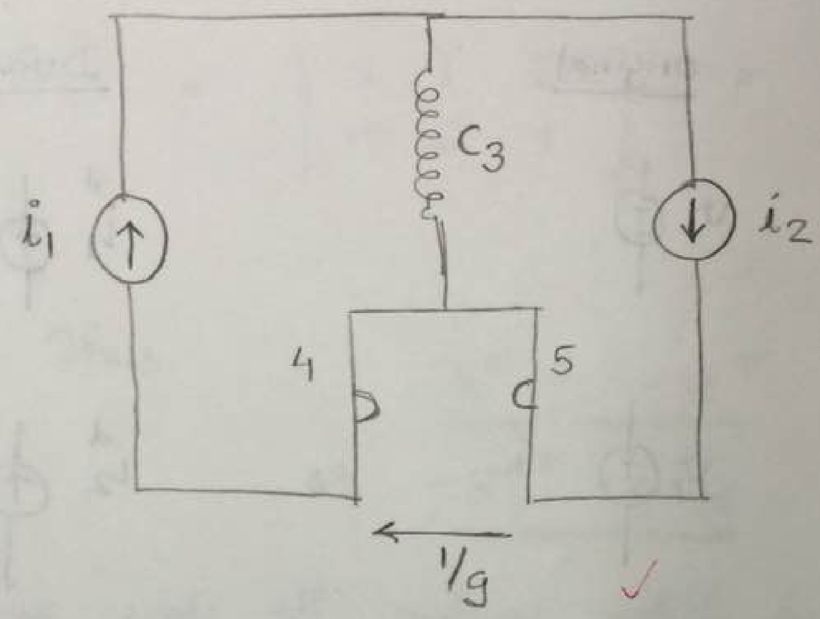
tie set matrix $J' = \left[\begin{array}{ccc|cc} 1 & 1 & 0 & 1 & 0 \\ -1 & 0 & 1 & 0 & 1 \end{array}\right]$

compare e, J with e' J'

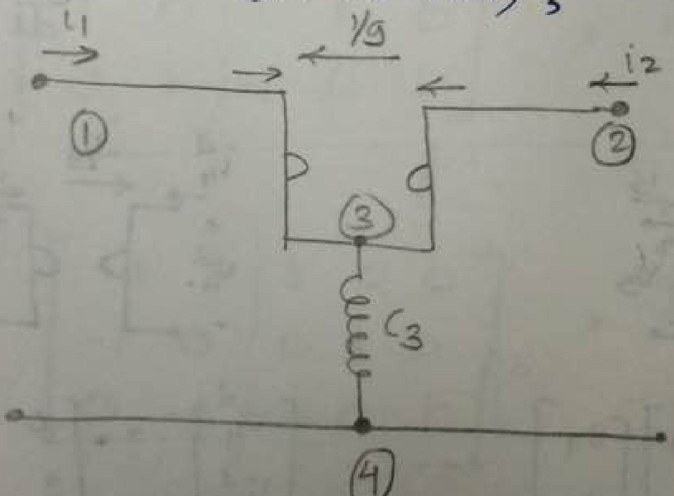
$e = [\, I_2 \mid K \,]$ $\quad e' = [\, I_3 \mid -K^T \,]$

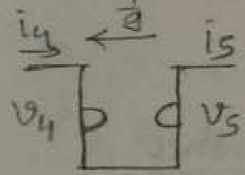
$J = [\, -K^T \mid I_3 \,]$ $\quad J' = [\, K \mid I_2 \,]$

AKSHAY SINGH



(d) Forming a two-port network, (for the dual),





$$\begin{bmatrix} i_4 \\ i_5 \end{bmatrix} = \begin{bmatrix} 0 & 1/g \\ -1/g & 0 \end{bmatrix} \begin{bmatrix} v_4 \\ v_5 \end{bmatrix}$$

$$i_1 = +\frac{1}{g}(v_2 - v_3) = +\frac{1}{g}v_2 - \frac{1}{g}v_3$$

$$i_2 = -\frac{1}{g}(v_1 - v_3) = -\frac{1}{g}v_1 + \frac{1}{g}v_3$$

$$i_3 = -\frac{1}{g}(v_2 - v_3) + \frac{1}{g}(v_1 - v_3) - \frac{1}{sC_3}(v_4 - v_3)$$

$$= +\frac{1}{g}v_1 - \frac{1}{g}v_2 + \frac{1}{sC_3}v_3 - \frac{1}{sC_3}v_4$$

$$i_4 = -\frac{1}{sC_3}(v_3 - v_4) = -\frac{1}{sC_3}v_3 + \frac{1}{sC_3}v_4$$

$$\begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{g} & -\frac{1}{g} & 0 \\ -\frac{1}{g} & 0 & \frac{1}{g} & 0 \\ \frac{1}{g} & -\frac{1}{g} & \frac{1}{sC_3} & -\frac{1}{sC_3} \\ 0 & 0 & -\frac{1}{sC_3} & \frac{1}{sC_3} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix}$$

Indefinite Admittance matrix

(Not asked for in the question)

Now setting $v_4 = 0$

$i_4 = i_1 + i_2 + i_3$

$$\begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 0 & 1/g & -1/g \\ -1/g & 0 & 1/g \\ 1/g & -1/g & 1/sC_3 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$$

To eliminate i_3, set $i_3 = 0$

$$i_3 = \frac{v_1}{g} - \frac{v_2}{g} + \frac{v_3}{sC_3} = 0$$

$$\Rightarrow v_3 = \frac{sC_3}{g}(v_1 - v_2)$$

$$\therefore \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} \frac{-sC_3}{g^2} & \frac{sC_3}{g^2} + \frac{1}{g} \\ \frac{sC_3}{g^2} - \frac{1}{g} & -\frac{sC_3}{g^2} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$