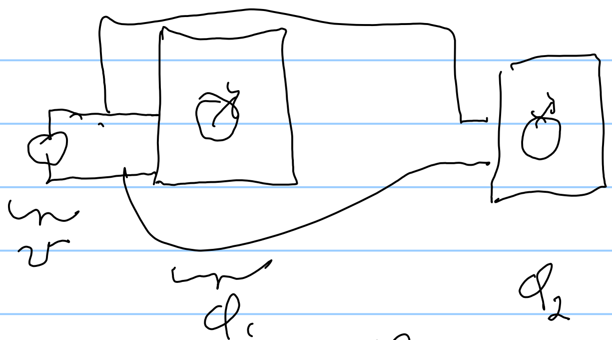
Note Title 10/27/2016

EE610
10/27/16



$\langle v, \phi \rangle = \text{value}$

$\langle v, \cdot \rangle \approx v$

$$\langle v, \phi \rangle = \int_{-\infty}^{\infty} v(t)\phi(t)\,dt,$$

ϕ are ∞'ly differentiable with compact support

testing functions $\in \mathcal{D}$

$\langle v, \cdot \rangle$

dual space $= \mathcal{D}' =$ distributions

$$\delta(t) \Rightarrow \int_{-\infty}^{\infty} \delta(t)\phi(t)\,dt = \phi(0) \Rightarrow \langle \delta, \phi \rangle = \phi(0)$$

$$\langle f(t), e^{-st} \rangle = \int_{-\infty}^{\infty} f(t) e^{-st}\, dt \Rightarrow \text{for } f(t) = \delta(t) \Rightarrow \langle \delta(t), e^{-st} \rangle = e^{0t} = 1$$

$$\mathcal{L}[f(t)] = \langle f(t), e^{-st} \rangle, \quad \mathcal{L}[\delta] = 1$$

$\shortparallel$

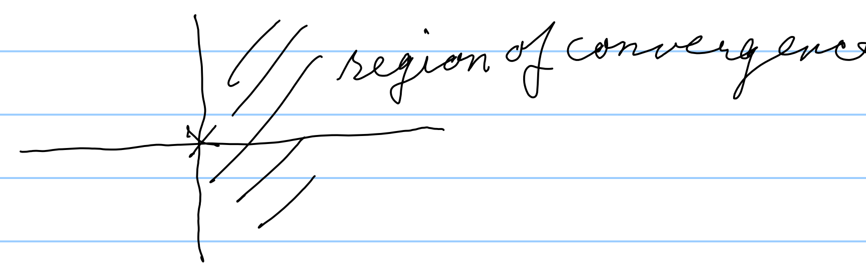
2-sided Laplace transform

$$f(t) = u(t), \quad u(t) = \begin{cases} 1 & t > 0 \\ 0 & t < 0 \end{cases} \Rightarrow \int_{-\infty}^{\infty} u(t) e^{-st}\, dt = \int_0^{\infty} e^{-st}\, dt$$

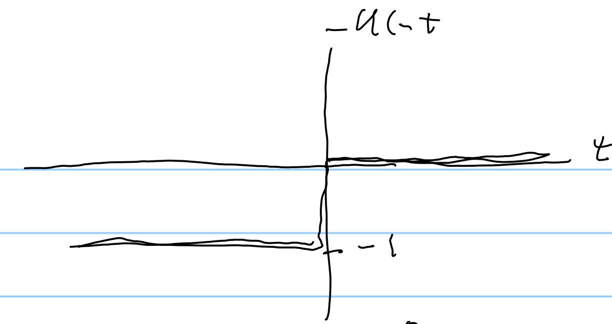
$$= -\frac{1}{s} e^{-st}\Big|_{t=0}^{\infty} = \frac{1}{s} - \underbrace{\frac{e^{-s\infty}}{s}}_{\text{exists if } \operatorname{Re} s = \sigma > 0} = \frac{1}{s} \text{ if } \sigma > 0$$

$$s = \sigma + j\omega$$

Region of convergence is $\sigma \geq 0$



Look at $-u(-t)$

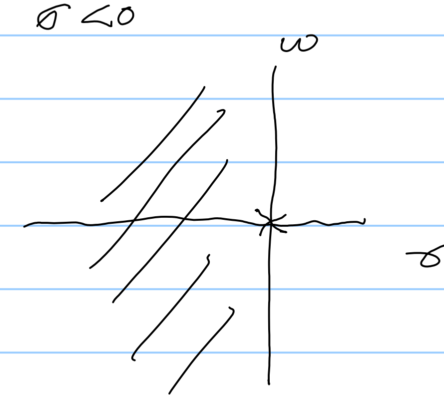


$$\mathcal{L}(-u(-t)) = \int_{-\infty}^{\infty} -u(-t)e^{-st}\,dt = -\int_{-\infty}^{0} 1 \cdot e^{-st}\,dt = -\left(\frac{-1}{s}\right)e^{-st}\Big|_{-\infty}^{0}$$

$$= \frac{1}{s} - \underbrace{\frac{1}{s}e^{-(-\infty)s}} \qquad = \frac{1}{s} \text{ if } \sigma < 0$$

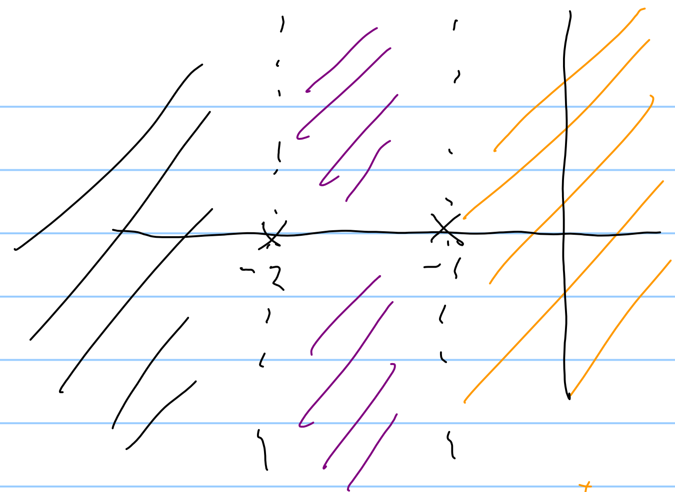
converges if $\mathrm{Re}\, s < 0$

LHP



If $\mathcal{L}[f(t)] = \dfrac{1}{s+1} + \dfrac{1}{s+2}$

has 3 regions of convergence



$-e^{-t}u(-t)$
$-e^{-2t}u(-t)$

$e^{-t}u(t)$
$+e^{-2t}u(t)$

$e^{-2t}u(t)$
$-e^{-t}u(-t)$