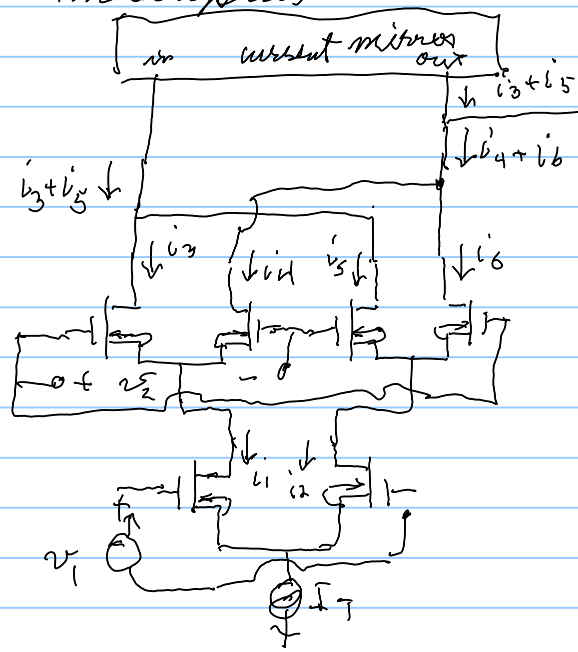
Note Title 10/17/2016

EE 610

10/20/16

Gilbert multiplier



$(i_4+i_6)-(i_3+i_5)$

$i_3 = \frac{i_1}{2}\left(1+f(v_2)\right)$

$i_4 = \frac{i_1}{2}\left(1-f(v_2)\right)$

$i_5 = \frac{i_2}{2}\left(1-f(v_2)\right)$

$i_6 = \frac{i_2}{2}\left(1+f(v_2)\right)$

$i_1 = \frac{I_T}{2}\left(1+f(v_1)\right)$

$i_2 = \frac{I_T}{2}\left(1-f(v_1)\right)$

$$i_3 = \frac{I_T}{4}\left(1+f(v_1)\right)\left(1+f(v_2)\right) = \frac{I_T}{4}\left(1+f(v_1)+f(v_2)+f(v_1)f(v_2)\right)$$

$$i_4 = \frac{I_T}{4}\left(1+f(v_1)\right)\left(1-f(v_2)\right) = \frac{I_T}{4}\left(1+f(v_1)-f(v_2)-f(v_1)f(v_2)\right)$$

$$i_5 = \frac{I_T}{4}\left(1-f(v_1)\right)\left(1-f(v_2)\right) = \frac{I_T}{4}\left(1-f(v_1)-f(v_2)+f(v_1)f(v_2)\right)$$

$$i_6 = \frac{I_T}{4}\left(1-f(v_1)\right)\left(1+f(v_2)\right) = \frac{I_T}{4}\left(1-f(v_1)+f(v_2)-f(v_1)f(v_2)\right)$$

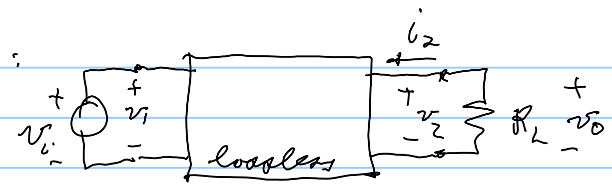
$$i_3+i_5 = \frac{I_T}{4}\left(2+2f(v_1)f(v_2)\right), \quad -(i_4+i_6) = -\frac{I_T}{4}\left(2-2f(v_1)f(v_2)\right)$$

$$(i_3+i_5)-(i_4+i_6) = \frac{I_T}{4}\left(4f(v_1)f(v_2)\right) = I_T f(v_1)f(v_2)$$

near origin $\approx I_T \cdot (a)\, v_1 v_2 \sqrt{1-bv_1^2}\sqrt{1-bv_2^2}$

gives a very good multiplier

Synthesis of $\frac{v_0}{v_i}$:



$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}, \quad \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} v_i \\ v_0 \end{bmatrix}; \quad v_0 = -R_L i_2, \quad G_L = 1/R_L.$$

$$\begin{bmatrix} i_1 \\ -G_L v_0 \end{bmatrix} = \begin{bmatrix} y_{11} v_i + y_{12} v_0 \\ y_{21} v_i + y_{22} v_0 \end{bmatrix} \Rightarrow (-G_L - y_{22}) v_0 = y_{21} v_i$$

$$\frac{v_0}{v_i} = \frac{-y_{21}}{G_L + y_{22}} = \frac{-y_{21}/G_L}{1 + y_{22}/G_L} = \frac{-\tilde{y}_{21}}{1 + \tilde{y}_{22}}$$

If $\tilde{y}_{22}(s) = \frac{N_{22}(s)}{D_{22}(s)}$ then $1 + \tilde{y}_{22} = \frac{N_{22}(s) + D_{22}(s)}{D_{22}(s)}$

$$\frac{v_o}{v_i} = \frac{-y_{21}(s) \cdot D_{22}(s)}{N_{22}(s) + D_{22}(s)} = \frac{K}{P(s)} \qquad P(s) = \text{Hurwitz}$$

if low-pass

create $y_{22} = \dfrac{\text{Ev } P(s)}{\text{Od } P(s)}$ then y_{22} is an lossless PR function

also if passive all poles of y_{21} are in y_{22} & if lowpass all zeros of $D_{22}(s)$ are poles of y_{21} to get the numerator as a constant: If low-pass synthesize y_{22} by 1st Cauer

Ex: $$\frac{v_o}{v_i}(s) = \frac{K}{s^6 + s^5 + 9s^4 + 6s^3 + 23s^2 + 8s + 15}$$

$$= \frac{K/(s^5 + 6s^3 + 8s)}{1 + \dfrac{(s^6 + 9s^4 + 23s^2 + 15)}{s^5 + 6s^3 + 8s}}$$

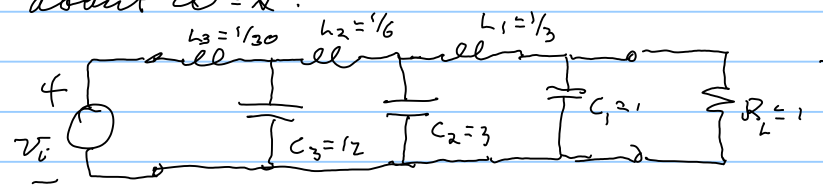
$$\Rightarrow \quad y_{22}(s) = \frac{s^6 + 9s^4 + 23s^2 + 15}{s^5 + 6s^3 + 8s}$$

1st Cauer

$$s^5+6s^3+8s \;\Big|\; s^6+9s^4+23s^2+15 \quad \to s$$
$$s^6+6s^4+8s^2$$
$$3s^4+15s^2+15 \;\Big|\; s^5+6s^3+8s \quad \to \tfrac{1}{3}s$$
$$s^5+5s^3+5s$$
$$s^3+3s \;\Big|\; 3s^4+15s^2+15 \quad \to 3s$$
$$3s^4+9s^2$$
$$6s^2+15 \;\Big|\; s^3+3s \quad \to \tfrac{1}{6}s$$
$$s^3+\tfrac{5}{2}s$$
$$\tfrac{1}{2}s \;\Big|\; 6s^2+15 \quad \to 12s$$
$$6s^2$$
$$15 \;\Big|\; \tfrac{1}{2}s \quad \to \tfrac{1}{30}s$$

$$y_{22}(s) = s + \cfrac{1}{\tfrac{1}{3}s + \cfrac{1}{3s + \cfrac{1}{\tfrac{1}{6}s + \cfrac{1}{12s + \cfrac{1}{\tfrac{1}{30}s}}}}} \quad \begin{matrix}\leftarrow y\\ \leftarrow z\\ \leftarrow y\\ \leftarrow z\\ \leftarrow y\\ \leftarrow z\end{matrix}$$

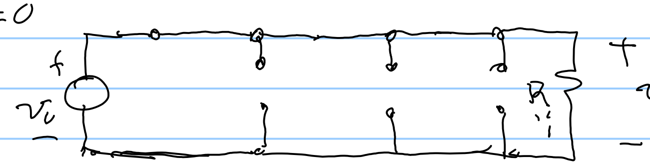
= continued fraction expansion about $\infty = s$.



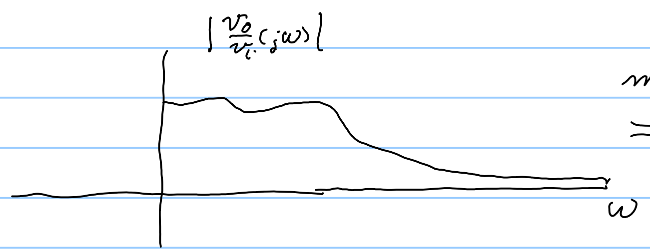
$$\Rightarrow \frac{v_o}{v_i} = \frac{K}{s^6 + \dots + 8s + 15}$$

$$v_o/v_i\,(s=0) = \frac{K}{15}$$

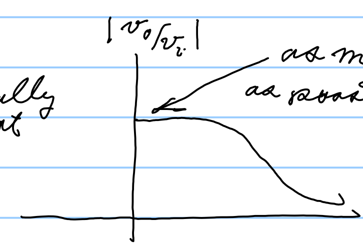
at $s=0$



$v_o = v_i \quad \left.\frac{v_o}{v_i}\right|_{s=0} = 1 = \frac{K}{15}, \quad K=15$



maximally flat $\Rightarrow$



as many derivatives as possible $=0$

Given that $T(s) = v_o/v_i$ = low-pass maximally flat of degree n

$$\left.\frac{d^k|T(j\omega)|}{d\omega^k}\right|_{\omega=0} \quad ; \quad T(s) = \frac{K}{s^n + a_{n-1}s^{n-1} + \dots + a_1 s + a_0}$$

$$d\left(1/|T(j\omega)|\right)/d\omega \Rightarrow d\left(\frac{1}{T(j\omega)T^*(j\omega)}\right)\Big/d\omega = \frac{d}{d\omega}\left(\frac{1}{T(j\omega)}\cdot\frac{1}{T(-j\omega)}\right)$$

$$\frac{K^2}{T(j\omega)T(-j\omega)} = \omega^{2n} + b_{2n-2}\,\omega^{2n-2} + \dots + b_2\,\omega^2 + b_0$$

$$\frac{d\;{}^1/_{f(\omega)}}{d\omega} = -\frac{1}{(f(\omega))^2}\cdot\frac{df(\omega)}{d\omega}$$

$$\frac{d\left({}^{K^2}/_{T(j\omega)T^*(j\omega)}\right)}{d\omega} = 2n\,\omega^{2n-1} + 2n-2\,b_{2n-2}\,\omega^{2n-3} + \dots\ 2b_2\,\omega^1$$

$\vdots$