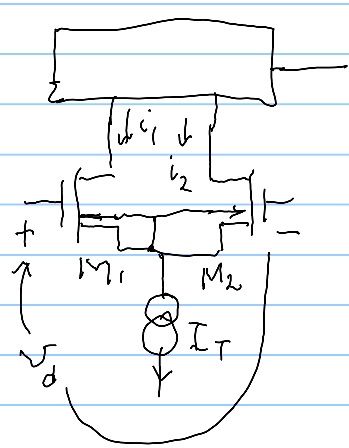
Note Title 10/17/2016

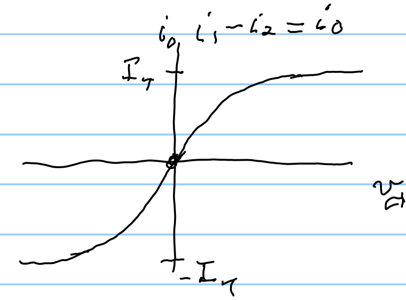
EE610
10/18/16
corrected

Differential pair - NMOS



$i_o = i_1 - i_2$

$i_1 + i_2 = I_T$



assume M_1 & M_2 are in saturation (or off)

$$i_d = \beta\left(v_{GS} - V_{th}\right)^2 \qquad \beta = \frac{KP}{2}\cdot\frac{W}{L}, \quad KP = \mu \cdot C_{ox}$$

$$i_1 = \beta\left(v_{G_1S} - V_{th}\right)^2, \quad i_2 = \beta\left(v_{G_2S} - V_{th}\right)^2$$

$$\sqrt{i_1/\beta} = v_{G_1S} - V_{th} \qquad \sqrt{i_2/\beta} = v_{G_2S} - V_{th}$$

$$v_d = v_{G_1S} - v_{G_2S} = \left(v_{G_1S} - V_{th}\right) - \left(v_{G_2S} - V_{th}\right) = \sqrt{i_1/\beta} - \sqrt{i_2/\beta} \Rightarrow \text{square}$$

$$v_d^2 = i_1/\beta + i_2/\beta - 2\frac{1}{\beta}\cdot\sqrt{i_1 i_2} \Rightarrow \sqrt{i_1 i_2} = \frac{1}{2}\left[i_1 + i_2 - \beta v_d^2\right]$$

$$i_2 = I_T - i_1 \quad \Rightarrow \sqrt{i_1(I_T - i_1)} = \frac{1}{2}\left[i_1 + I_T - i_1 - \beta v_d^2\right] = \frac{1}{2}\left(I_T - \beta v_d^2\right)$$

square again $-i_1^2 + I_T i_1 = \frac{1}{4}\left(I_T - \beta v_d^2\right)^2$

$$i_1^2 - I_T i_1 + \frac{1}{4}\left(I_T - \beta v_d^2\right)^2 = 0$$

$$i_1 = \frac{I_T}{2} \pm \frac{1}{2}\sqrt{\underbrace{I_T^2 - 4\cdot\frac{1}{4}\left(I_T - \beta v_d^2\right)^2}}$$

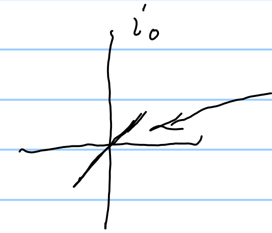
$$I_T^2 - \left(I_T^2 - 2\beta I_T v_d^2 + \beta^2 v_d^4\right) = 2\beta I_T v_d^2 - \beta^2 v_d^4 = 2\beta I_T v_d^2\left(1 - \frac{\beta}{2I_T} v_d^2\right)$$

$$= \frac{I_T}{2} \pm \frac{1}{2} I_T v_d \sqrt{\frac{2\beta}{I_T}}\sqrt{1 - \frac{\beta}{2I_T}\cdot v_d^2}$$

$$= \frac{I_T}{2}\left(1 + \sqrt{2\beta/I_T}\cdot v_d\sqrt{1 - \left(\frac{\beta}{2I_T}\right)v_d^2}\right) = \frac{I_T}{2} + \underbrace{\sqrt{\frac{\beta}{2I_T}}}_{\substack{\text{linear} \\ = G_m}} v_d \sqrt{1 - \left(\frac{\beta}{2I_T}\right)v_d^2}$$

$$i_2 = \frac{I_T}{2}\left(1 - \sqrt{\frac{2\beta}{I_T}}\, v_d \sqrt{1 - \left(\beta/2I_T\right)v_d^2}\right)$$

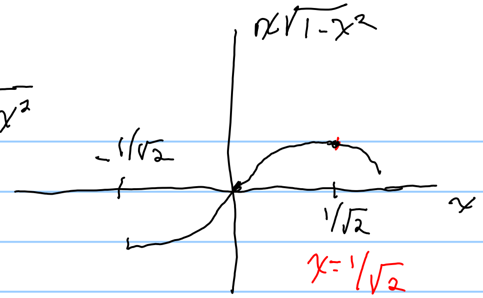
$$i_o = i_1 - i_2 = \frac{2I_T}{2}\sqrt{\frac{2\beta}{I_T}}\, v_d \cdot \sqrt{1 - \left(\beta/2I_T\right)v_d^2}$$



G_m for an OTA $= \sqrt{2 I_T \beta}$

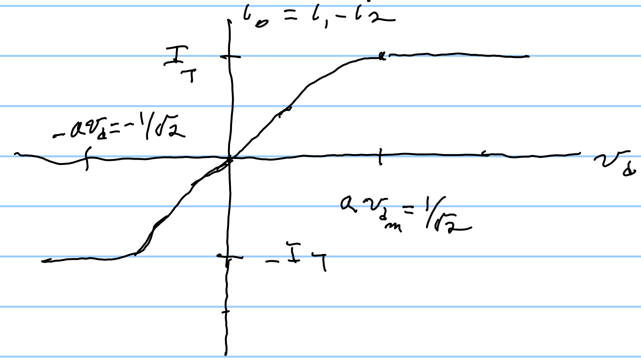
desire max of $\frac{1}{2I_T}\left(a v_d\sqrt{1 - a^2 v_d^2}\right) \Rightarrow x\sqrt{1-x^2}$; max x, $a = \sqrt{\beta/2I_T}$

$x\sqrt{1-x^2}$



$$\frac{d\, x\sqrt{1-x^2}}{dx} = \sqrt{1-x^2} + x\left(\frac{+1/2}{\sqrt{1-x^2}} \cdot (-2x)\right)$$

$$= \frac{(1-x^2) - x^2}{\sqrt{1-x^2}} = 0 \Rightarrow 2x^2 = 1$$



If M_1 & M_2 are in subthreshold $i_d \approx I_D e^{v_{GS}/nV_T}$, $V_T = \dfrac{KT}{|q|}$

$$i_1 = I_D e^{v_{G_1S}/(nV_T)}, \quad i_2 = I_D e^{v_{G_2S}/(nV_T)}$$

$$\ln(i_1/I_D) = v_{G_1S}/nV_T, \quad \ln(i_2/I_D) = v_{G_2S}/nV_T; \quad \frac{v_{G_1S} - v_{G_2S}}{nV_T} = \frac{v_d}{nV_T}$$

$$\frac{v_d}{nV_T} = \ln(i_1/I_D) - \ln(i_2/I_D) = \ln(i_1/i_2)$$

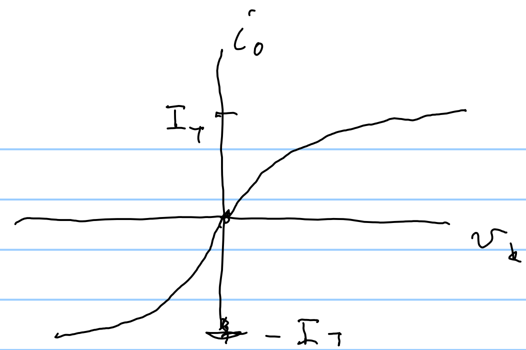
$$\frac{i_1}{i_2} = e^{v_d/nV_T} \qquad i_1 + i_2 = I_T \Rightarrow e^{v_d/nV_T} i_2 + i_2 = I_T$$

$$i_2 = \frac{I_T}{1 + e^{v_d/nV_T}} = \frac{I_T}{e^{v_d/2nV_T}\left(e^{-v_d/nV_T 2} + e^{+v_d/nV_T 2}\right)}$$

$$i_1 = e^{v_d/nV_T} i_2 = \frac{e^{v_d/2nV_T} I_T}{2\cosh(v_d/2nV_T)}$$

$$i_0 = i_1 - i_2 = \frac{I_T\left(e^{\frac{v_d}{2nV_T}} - e^{-v_d/2nV_T}\right)}{2\cosh(v_d/2nV_T)} = I_T \frac{\sinh\left(\frac{v_d}{2nV_T}\right)}{\cosh\left(\frac{v_d}{2nV_T}\right)}$$

$$i_0 = I_T \tanh\left(\frac{v_d}{2nV_T}\right)$$



$$\frac{di_o}{dv_d} = I_T\left(1-\tanh^2\left(\frac{v_d}{2nV_T}\right)\right)\cdot\frac{1}{2nV_T}$$

$$= \frac{I_T}{2nV_T}\cdot\left(1-\tanh^2\left(\frac{v_d}{2nV_T}\right)\right)$$

$\Rightarrow$ slope near origin $\approx G_m = \dfrac{I_T}{2nV_T}$