EE610

10/13/16

Note Title 10/13/2016

given $y(s) = \frac{N(s)}{D(s)}$ then it comes from a differential equation, linear time-invariant

$$a_n \frac{d^n i(t)}{dt^n} + a_{n-1}\frac{d^{n-1} i(t)}{dt^{n-1}} + \dots + a_1 \frac{d i(t)}{dt} + a_0 i(t) = b_m \frac{d^m v(t)}{dt^m} + \dots + b_1 \frac{d v(t)}{dt} + b_0 v(t)$$

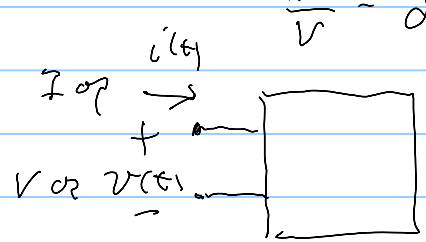
if $v(t) = V e^{st}$ then $i(t) = I e^{st}$, where $-\infty < t < \infty$

$$\left[a_n s^n I + a_{n-1} s^{n-1} I + \dots a_1 s I + a_0 I\right] e^{st} = \left[b_m s^m V + \dots b_1 s V + b_0 V\right] e^{st}$$

as $e^{st} \neq 0$ can cancel

$$\left[a_n s^n + \dots a_0\right] I = \left[b_m s^m + \dots b_0\right] V$$

$$\frac{I}{V} = y(s) = \frac{b_m s^m + \dots + b_0}{a_n s^n + \dots + a_0} = \frac{N(s)}{D(s)}$$



if $I = 0$ then we $N(s)\, v(t) = 0$
$i(t) = 0$

$\therefore$ if $i(t)=0$, open circuit & $v(t)\neq 0$ if $N(s)=0$

the zeros of $N(s)$ give open circuit natural frequencies

a zero of N goes with $v(t)=Ve^{s_0 t}$, s_0 = a natural frequency

$\overset{\shortparallel}{s_0}$

if $v(t)=0$ then $i(t)=Ie^{st}=ID(s)e^{st}=0$ if $I=0$ or $D(s)=0$

$\Rightarrow$ zeros of $D(s)$ allow nonzero $i(t)$ when $v(t)=0$

$\overset{\shortparallel}{s_0}$ are short circuit natural frequencies

$$y(s)=\frac{2s(s^2+5)}{(s^2+4)(s^2+9)}=\frac{I}{V}(s)\Rightarrow(2s^3+10s)Ve^{st}=(s^4+13s^2+36)Ie^{st}$$

$$2\frac{d^3v(t)}{dt^3}+10\frac{dv(t)}{dt}=\frac{d^4i(t)}{dt^4}+13\frac{d^2i(t)}{dt^2}+36i(t)$$

if $i(t)=0$ then $(2s^3+10s)v(t)=0=N(s)v(t)=s(s^2+10)v(t)$

if e^{st} has $s=0$ then $N(s)e^{0t} = 0$

if e^{st} has $s = \pm j\sqrt{10}$ then $N(s)\Big|_{s=j\sqrt{10}} e^{j(+\sqrt{10})t} = 0$ & $N(s)\Big|_{s=-j\sqrt{10}} e^{-j\sqrt{10}\,t}$

From the differential equation

$$2\frac{d^3v(t)}{dt^3} + 10\frac{dv(t)}{dt} = 0$$

has solutions

$$v(t) = \left[A_0 e^{0t} + B e^{+j\sqrt{10}t} + B^* e^{-j\sqrt{10}t}\right]\Big|_{0<t}$$

$$v(t) = A_0 + (B\cos(\sqrt{10}t) + jB\sin(\sqrt{10}\,t)) + B^*\cos(-\sqrt{10}t) + jB^*(\sin(-\sqrt{10}\,t))$$

Van der Pol oscillator

$$\ddot{x} + \epsilon(x^2-1)\dot{x} + \omega_0^2 x = 0$$

if $\epsilon = 0$ then $e^{\pm j\omega t}$ gives a solution

if $x^2 < 1 \Rightarrow$ the $\dot{x}$ coefficient is < 0 & it represents negative resistance

if $x^2 > 1 \Rightarrow$ " " " " > 0 & " " positive resistance

$$\ddot{x} + \frac{d}{dt}F(x) + \omega_0^2 x = 0$$

$y = \dot{x} + F(x)$; $\dot{} = \frac{d}{dt}$

$$\frac{dy}{dt} = \ddot{x} + \frac{dF(x)}{dt} = -\omega_0^2 x$$

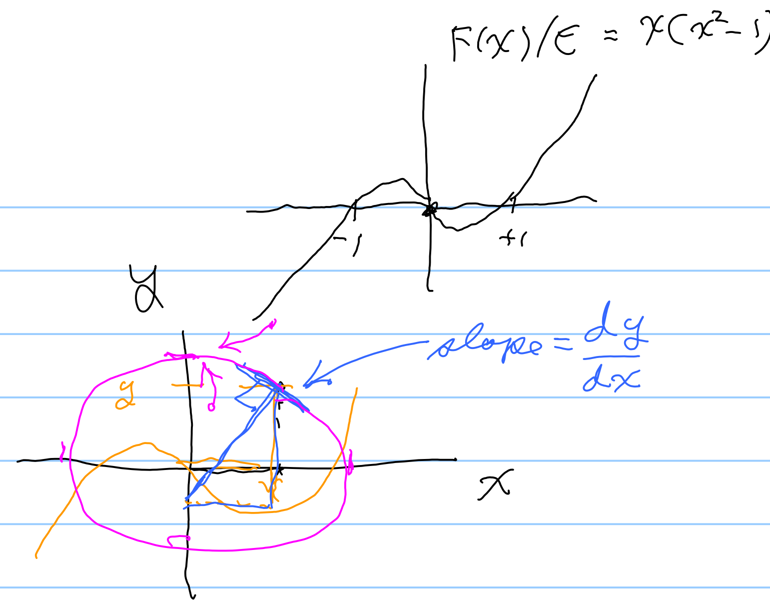
$$\frac{dx}{dt} = y - F(x)$$

; $\frac{dF(x)}{dx} = \epsilon(x^2-1) \Rightarrow \epsilon\left(\frac{1}{3}x^3 - x\right)$

$$\frac{dy}{dt} = -\omega_0^2 x$$

$$\frac{dx}{dt} = y - \epsilon \frac{1}{3}(x^3 - x)$$

$$\frac{dy}{dt} = -\omega_0^2 x$$



$$\frac{dy}{dx} = \frac{-\omega_0^2 x}{y - F(x)}$$

$$= \frac{\omega_0^2 x}{F(x) - y}$$

depending on ϵ is the type of oscillation (gives an y vs x circle for small ϵ)

ϵ small get sine waves at ω_0 frequency.
ϵ large get relaxation (square wave) oscillations

and we can replace $F(x)$ by reasonable odd functions