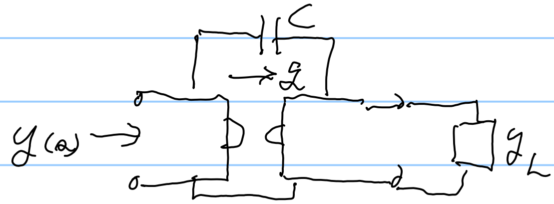
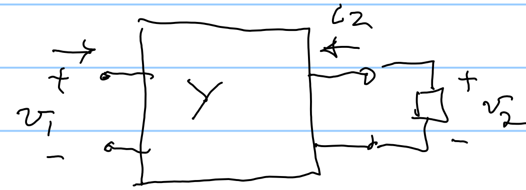
Note Title 10/11/2016

EE 610
10/11/10

Richards' function, p. 361 (for Bott-Duffin synthesis)

$$R_z(s) = \frac{k z(s) - s z(k)}{k z(k) - s z(s)} \quad ; \quad R_{y(s)}(s) = \frac{k y(k) - s y(s)}{k y(s) - s y(k)}$$





$i_2 = -y_L v_2$

$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} i_1 \\ -y_L v_2 \end{bmatrix}$$

$$-y_L v_2 = y_{21} v_1 + y_{22} v_2 \Rightarrow (y_L + y_{22}) v_2 = -y_{21} v_1$$

$$v_2 = -(y_L + y_{22})^{-1} y_{21} v_1$$

$$i_1 = y_{11} v_1 + y_{12} v_2 = y_{11} v_1 - y_{12} (y_L + y_{22})^{-1} y_{21} v_1 \; ; \; \Delta Y = \text{determinant of } Y$$

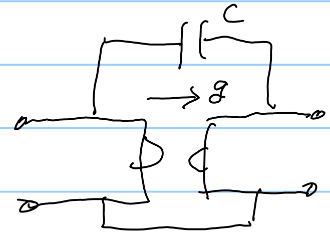
$$= \frac{\left[y_{11}(y_L + y_{22}) - y_{12} y_{21} \right]}{y_L + y_{22}} v_1 \Rightarrow y = \frac{i_1}{v_1} = \frac{y_{11} y_L + \Delta y}{y_L + y_{22}}$$

$$\Rightarrow y y_L + y y_{22} = y_{11} y_L + \Delta Y \Rightarrow (y - y_{11}) y_L = \Delta y - y_{22} y$$

$$y_L(a) = \frac{\Delta Y - y_{22}\, y(a)}{y - y_{11}}$$

$$R_{g(a)}(a) = \frac{k y(k) - a y(a)}{k y(a) - a y(k)} = \frac{k y(k)}{k} \left[\frac{1 - \frac{a y(a)}{k y(k)}}{y(a) - \frac{a y(k)}{k}} \right]$$

Y for



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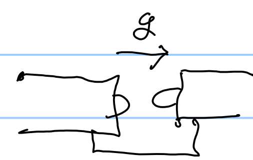

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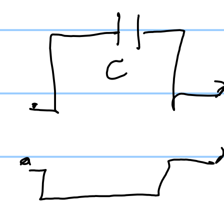


$$Y = \begin{bmatrix} aC & -aC - g \\ -aC + g & aC \end{bmatrix}$$

$$Y_g = \begin{bmatrix} 0 & -g \\ g & 0 \end{bmatrix} , \quad Y_\pi = \begin{bmatrix} aC & -aC \\ -aC & aC \end{bmatrix}$$

$$\Delta Y = (\alpha c)^2 - (-\alpha c - g)(-\alpha c + g) = (\alpha c)^2 - (\alpha c + g)(\alpha c - g) = (\alpha c)^2 - (\alpha c)^2 - (-g^2)$$
$$= g^2$$

$$y_L(\alpha) = \frac{\Delta Y - Y_{22}(\alpha)\, y(\alpha)}{y(\alpha) - y_{11}(\alpha)} = \frac{g^2 - \alpha c\, y(\alpha)}{y(\alpha) - \alpha c} = g^2 \left[\frac{1 - \frac{\alpha c\, y(\alpha)}{g^2}}{y(\alpha) - \alpha c} \right]$$

compare to

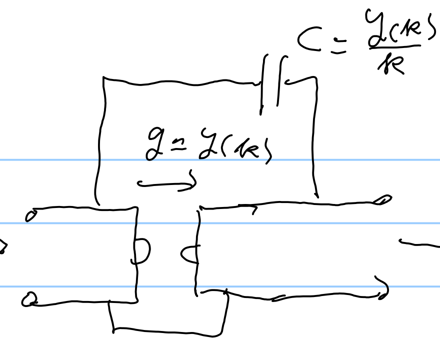
$$R_y(\alpha) = y(k) \left[\frac{1 - \frac{\alpha\, y(\alpha)}{k\, y(k)}}{y(\alpha) - \frac{\alpha\, y(k)}{k}} \right] \quad \Rightarrow \quad \frac{\alpha\, y(\alpha)}{k\, y(k)} = \frac{c}{g^2} \alpha\, y(\alpha) \Rightarrow k\, y(k) = \frac{g^2}{c}$$

$$\Rightarrow \alpha c = \alpha \frac{y(k)}{k} \Rightarrow c = \frac{y(k)}{k}$$

$$\Rightarrow k\, y(k) = \frac{g^2}{c} = \frac{k}{y(k)} \cdot g^2 \Rightarrow g^2 = y^2(k) \Rightarrow g = \pm y(k)$$

$$\frac{y_L(\alpha)}{y(k)} = y(k) \left[\frac{1 - \frac{\alpha c\, y(\alpha)}{g^2}}{y(\alpha) - \alpha c} \right] = R_y(\alpha)$$

need $g = + y(k)$ if $k > 0$



$Y(s) \rightarrow$ $\rightarrow Y_L = Y(k)\, R_Y(s)$ as $\dfrac{Y_L}{Y(k)} = R_Y(s)$

is PR if Y is PR if $k > 0$ as $R_Y(s)$ is a Richards' function

$$Y_L(s) = Y(k) \cdot \frac{(s-k)}{s-k} \cdot Y_2(s)$$

$$R_{Y(s)}(s) = \frac{kY(k) - sY(s)}{kY(s) - sY(k)}$$

if $Y(s) = -Y(s)$ then

then for $s = -k$

$$\frac{kY(k) + kY(-k)}{kY(-k) - (-k)Y(k)} = \frac{kY(k) - kY(k)}{} = 0$$

$$kY(-k) - (-k)Y(k) = 0$$

$$\frac{(Y(s) + Y(-s))}{2} = Ev\, Y(s)$$

for lossless PR function $y(s) = -y(-s) \Rightarrow$ can use any real k
and degree of y_L is 1 less than the degree of y

Ex: $y(s) = \dfrac{s(s^2+3)}{s^2+2}$ choose $k=1$, $y(1) = y(k) = \dfrac{1(1+3)}{1+2} = 4/3$

$$\frac{y_L(s)}{y(k)} = \frac{y_L}{4/3} = \frac{k\,y(k) - s\,y(s)}{k\,y(s) - s\,y(k)} = \frac{1 \cdot 4/3 - s\left(s\left(\frac{s^2+3}{s^2+2}\right)\right)}{1 \cdot s\left(\frac{s^2+3}{s^2+2}\right) - s \cdot 4/3}$$

$$= \frac{4/3(s^2+2) - (s^4+3s^2)}{s^3+3s - \frac{4}{3}(s^3+2s)} = \frac{-s^4 + \left(\frac{4}{3}-3\right)s^2 + 8/3}{\left(1-4/3\right)s^3 + \left(3-\frac{8}{3}\right)s}$$

$$= \frac{-3s^4 - 5s^2 + 8}{-s^3 + 1s}$$ and s^2-1 should divide out $= (s-k)(s+k)$

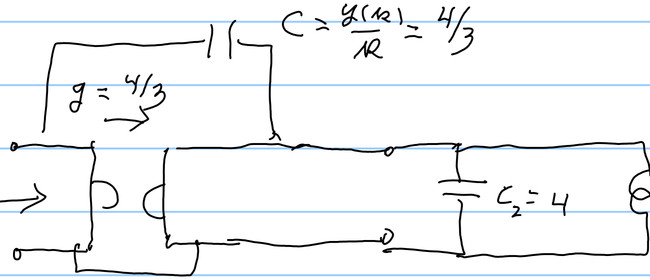
$$= \frac{(s^2-1)\cdot(-1)(3s^2+8)}{-s(s^2-1)} = \frac{3s^2+8}{s} = 3s + \frac{8}{s} \Rightarrow$$

$$\begin{array}{r} -3s^2-8 \\ s^2-1 \,\big)\, \overline{-3s^4-5s^2+8} \\ \underline{-3s^4+3s^2} \\ -8s^2+8 \\ \underline{-8s^2+8} \\ 0 \end{array}$$

$$\frac{y_L}{y(1)} = 3s + \frac{8}{s}$$

$$y_L(s) = \frac{4}{3}\left(3s + 8/s\right) = 4s + \frac{32}{3s}$$

$$y(s) = \frac{s(s^2+3)}{(s^2+2)} \longrightarrow$$



$$y(s) = \frac{s(s^2+3)}{s^2+2}$$

$$y(s) + y(-s) = \frac{s(s^2+3)}{s^2+2} + \frac{(-s)(s^2+3)}{(s^2+2)}$$

$\equiv 0$ & $k = 1$ is a

zero of $Ev(y(s))$